

ALGEBRA 31

Description

Students in Algebra 31 should be highly motivated to work at an accelerated pace. Algebra 31 provides an in depth exploration of concepts of relation and functions, while additional topics include an advanced study of powers and roots, expansion, and application of systems of equations, probability, matrices, logarithmic and exponential functions, complex numbers, polynomial functions and their graphs, and quadratic relations and systems with an emphasis on problem solving and real-life applications. Additional units of study are sequences and series, linear programming, exponential growth, permutations, combinations, negative and fractional exponents, and the binomial theorem.

Course Overview

Course Goals

Students should:

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

Assessments

Common Assessments

Skill Assessments

Content Outline

- I. [Unit 1](#) - Algebraic Connections
- II. [Unit 2](#) - Algebraic Expressions, Equations and Inequalities
- III. [Unit 3](#) - Functions and Relations
- IV. [Unit 4](#) - Graphing
- V. [Unit 5](#) - Exponents, Radicals, and Logarithms
- VI. [Unit 6](#) - Polynomials

Standards

[State of Connecticut Mathematics Curriculum Frameworks](#)

Connecticut State Standards are met in the following areas:

- *Algebraic Reasoning: Patterns And Functions*
- *Numerical and Proportional Reasoning*
- *Working with Data: Probability and Statistics*

Grade Level Skills

Students will:

- Skills Matrix

Pacing Guide

Pacing Guide										
1st Marking Period			2nd Marking Period			3rd Marking Period		4th Marking Period		
September	October	November	December	January	February	March	April	May	June	
Unit 1	Unit 2		Unit 3			Unit 4		Unit 5		Unit 6
<u>Algebraic Connections</u>	<u>Algebraic Expressions, Equations and Inequalities</u>		<u>Functions and Relations</u>			<u>Graphing</u>		<u>Exponents, Radicals, and Logarithms</u>		<u>Polynomials</u>
3 weeks	7 ½ weeks		6 ½ weeks			6 weeks		5 weeks		5 weeks

Unit 1 - Algebraic Connections, 3 weeks [top](#)

Standards

Algebraic Reasoning: Patterns and Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technology.

1.1 Students should understand and describe patterns and functional relationships.

Core 1.1a Students should describe relationships and make generalizations about patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Core 1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Core 1.3a Students should manipulate equations, inequalities, and functions to solve problems.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technology.

2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships. .

Core 2.1a Students should extend the understanding of number to include integers, rational numbers, and real numbers.

2.1b Students should interpret and represent large sets of numbers with the aid of technology.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Core 2.2a Students should develop strategies for computation and estimation using properties of number systems to solve problems.

2.2b Students should solve proportional reasoning problems.

Working with Data: Probability and Statistics - Data can be analyzed to make informed decisions using a variety of strategies, tools and technology.

4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.

Core 4.1a Students should create the appropriate visual or graphical representation of real data.

4.2 Students should analyze data sets to form hypotheses and make predictions.

Core 4.2a Students should analyze real world problems using statistical techniques.

4.3 Students should understand and apply basic concepts of probability.

Core 4.3a Students should understand and apply the principles of probability in a variety of situations.

Extended 4.3a Students should solve problems using the methods of discrete mathematics.

Unit Objectives

Students will be able to:

- identify, describe, create and generalize numeric, geometric, and statistical patterns with tables, graphs, words, and symbolic rules.
- make and justify predictions based on patterns.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How can collecting, organizing and displaying data help us analyze information and make

Assessment

- Spring Jam

Skill Objectives

Students will:

- investigate the patterns and express the relationship between two variables in an equation

<ul style="list-style-type: none"> • identify the characteristics of functions and relations including domain and range. • model and solve problems with linear, inequalities, quadratic, and absolute value equations. • judge the reasonableness of the results of symbolic manipulations as related to authentic contexts. • analyze essential relations in a problem to determine possible functions that could model the situation. 	<p>reasonable predictions and informed decisions?</p> <p><u>Focus Questions</u></p> <ul style="list-style-type: none"> • How are matrices added, subtracted and multiplied? • How are inverse matrices used to solve linear systems? • How are counting principals applied to probability calculations? • How are binomial and normal distributions used? 	<ul style="list-style-type: none"> • find the theoretical and experimental probability of an event • use simulation as a method for estimating probability. • translate data into matrices and perform matrix addition and scalar multiplication. • perform matrix multiplication. • find the constraints or limitations of a real-world situation by using systems of inequalities. • solve a real-world problem using linear programming. • find the number of ways an event can occur using tree diagrams and the multiplication counting principle. • find the number of ways that items in a set can be arranged when all of the items are different and when some of the items are the same. • find the number of combinations of items without regard to order. • use the binomial theorem and Pascal's Triangle to expand binomials.
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Unit 2 – Algebraic Expressions, Equations and Inequalities, 7 ½ weeks [top](#)

Standards

Algebraic Reasoning: Patterns and Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technology.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Core 1.3a Students should manipulate equations, inequalities, and functions to solve problems.

1.1 Students should understand and describe patterns and functional relationships.

Extended 1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technology.

2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships. .

Core 2.1a Students should extend the understanding of number to include integers, rational numbers, and real numbers.

2.1b Students should interpret and represent large sets of numbers with the aid of technology.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Core 2.2a Students should develop strategies for computation and estimation using properties of number systems to solve problems.

2.2b Students should solve proportional reasoning problems.

Unit Objectives

Students will be able to:

- evaluate and interpret the graphs of linear, exponential, and polynomial functions.
- model and solve problems with linear, quadratic, and absolute value equations, and linear inequalities.
- determine equivalent representations of an algebraic equations or inequality to simplify and solve problems.
- judge the reasonableness of the results of symbolic manipulations as related to authentic contexts.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- use optimization strategies including linear programming.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?

Focus Questions

- How are the base ten number system and fractions, decimals, percents and ratios related?
- How are numerical and algebraic expressions evaluated and simplified?
- How are linear and absolute value equations and inequalities solved?
- How is algebra used to model and solve real life problems?
- How are operations performed on

Assessment

- Sum of an infinite series – (Problem with money; purchase of bonds on a regular basis)

Skill Objectives

Students will:

- solve equations algebraically, graphically and with technology.
- solve and graph one variable inequalities.
- solve and graph absolute value equations and inequalities.
- graph and solve linear and absolute value inequalities in two variables.
- solve linear systems by graphing.
- solve linear systems by substitution and linear combination.
- solve linear systems using matrices.

<ul style="list-style-type: none"> • apply the concepts of limits to sequences and asymptotic behavior of functions. 	<p>rational expressions and how are they simplified?</p> <ul style="list-style-type: none"> • How are rational equations graphed and solved? • How are variation and rational models used in real life situations? • How are sequences defined and their terms found? • How is summation notation used and how are the sums of arithmetic and geometric series found? 	<ul style="list-style-type: none"> • solve a system with 3 variables. • solve a system of linear inequalities by graphing. • find the next term in a sequence by looking for a pattern. Find the nth term of an arithmetic sequence. Find arithmetic means. • find sums of arithmetic series. Find specific terms in an arithmetic series. Use sigma notation to express sums. • find the nth term of a geometric sequence. Find geometric means. • find the sum of a geometric series. Use sigma notation to express sums. • find the sum of an infinite geometric series. • identify the field properties of our real number system. • identify the subsets of the real numbers. Simplify radical expressions. • define and use imaginary and complex numbers. • graph complex numbers and find their absolute value. Find the sum of complex numbers graphically. • solve a quadratic equation graphically and find the real roots. • solve a quadratic equation by factoring using GCF, difference of two squares, factoring a trinomial with leading coefficient of “a”. • solve a quadratic equation using the quadratic formula. Find the vertex of a parabola by using the equation $x = -b/(2a)$ • use the discriminate to determine the nature of the roots of quadratic equation. • solve quadratic-linear systems graphically and algebraically. • solve quadratic-quadratic systems graphically and algebraically. • add, subtract and multiply functions. • find the composition of functions.
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		<ul style="list-style-type: none">• find the inverse of a relation and determine if it is a function.• simplify rational expressions. Multiply and Divide rational expressions. Evaluate rational expressions. Factor sum and difference of cubes.• add and subtract rational expressions.• solve rational equations. Solve word problems that elicit rational equations such as work problems or rate problems.
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Unit 3 - Functions and Relations, 6 ½ weeks [top](#)

Standards

Algebraic Reasoning: Patterns and Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technology.

1.1 Students should understand and describe patterns and functional relationships.

Core 1.1a Students should describe relationships and make generalizations about patterns and functions.

Extended 1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Core 1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.

Extended 1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Core 1.3a Students should manipulate equations, inequalities, and functions to solve problems.

Extended 1.3a Students should use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technology.

2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships.

Core 2.1a Students should extend the understanding of number to include integers, rational numbers, and real numbers.

2.1b Students should interpret and represent large sets of numbers with the aid of technology.

Extended 2.1a Students should extend the understanding of number to include the set of complex numbers.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Core 2.2a Students should develop strategies for computation and estimation using properties of number systems to solve problems.

2.2b Students should solve proportional reasoning problems.

Unit Objectives

Students will be able to:

- determine equivalent representations of an algebraic equations or inequality to simplify and solve problems.
- judge the reasonableness of the results of symbolic manipulations as related to authentic contexts.
- analyze essential relations in a problem to determine possible functions that could model the situation.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?

Focus Questions

- How are equations of lines written?
- How are the distance and midpoint formulas used?

Assessment

- Students will create an equation and answer questions based on data. (Example: Hardly Harley page 97 present text)

Skill Objectives

Students will:

- define a function, domain, range and use function notation.
- determine if a function is linear and if it

<ul style="list-style-type: none"> • combine, compose, and invert functions. • solve systems of two linear equations using algebraic or graphical methods. • describe and compare properties and classes of linear, quadratic, and exponential functions. • recognize and explain the meaning of the slope and x- and y-intercepts as they relate to a context, graph, table or equation. 	<ul style="list-style-type: none"> • How are the equations of conic sections written, classified and graphed? • How are systems of quadratic equations solved? 	<p>is also a direct variation. Determine the slope of the linear function as a rate of change.</p> <ul style="list-style-type: none"> • write an equation of a line in slope-intercept form given the slope and one or two points. Write the equation of a line in point-slope form. Write the equation of a line that is parallel or perpendicular to a given equation. • write a quadratic function in the form $y = a(x-h)^2 + k$ by completing the square. • find the distance and midpoint between two points in the coordinate plane. Write equations of circles in standard form. Use completing the square to write equations of circles in standard form in order to find the center and radius. • write equations of ellipses in standard form. Graph ellipses with center at origin and center at (h,k). • write equations of hyperbolas in standard form. Graph hyperbolas with center at the origin and at (h,k). Graph rectangular hyperbolas of the form $xy = k$. • identify conic sections from their equation. • add, subtract and multiply functions. • find the composition of functions. • find the inverse of a relation and determine if it is a function.
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Unit 4 - Graphing, 6 weeks [top](#)

Standards

Algebraic Reasoning: Patterns and Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technology.

1.1 Students should understand and describe patterns and functional relationships.

Core 1.1a Students should describe relationships and make generalizations about patterns and functions.

Extended 1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Core 1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.

Extended 1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Core 1.3a Students should manipulate equations, inequalities, and functions to solve problems.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technology.

2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships.

Core 2.1a Students should extend the understanding of number to include integers, rational numbers, and real numbers.

2.1b Students should interpret and represent large sets of numbers with the aid of technology.

Unit Objectives

- Students will be able to:
- identify an appropriate symbolic representation for a function or relation displayed graphically or verbally.
 - explore conic sections and their applications graphically and symbolically.
 - judge the reasonableness of the results of symbolic manipulations as related to authentic contexts.
 - relate the graphical representation of a function family and find equations, intercepts, maximum or minimum values, asymptotes, and line of symmetry for that function.
 - recognize the effect of changes in

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?

Focus Questions

- How are ordered pairs, relations, functions, linear equations and inequalities in two variables, piecewise functions, and absolute value functions graphed?
- How are graphs and equations used to solve real life problems?
- How are algebraic methods used to

Assessment

- Real Estate Assessor or model a situation where a person is on a 50-foot cliff and is throwing a ball into the air. (Use the equation $h(t) = -4.9 t^2 + v_0 t + h_0$ feet . Use graphing calculator and develop questions.)

Skill Objectives

- Students will:
- graph a relationship between two sets of data, identify any trend, and describe any association. Find and use the equation of a trend line to make predictions.
 - find and use the equation of the linear regression line using technology.
 - solve and graph one variable inequalities.

<p>parameters on the graphs of functions.</p> <ul style="list-style-type: none"> represent functions and relations on the coordinate plane. 	<p>solve linear systems in two or three variables by graphing?</p> <ul style="list-style-type: none"> How are linear systems written and used to solve real life problems? How are quadratic equations solved? How are quadratic functions and inequalities graphed? 	<ul style="list-style-type: none"> solve and graph absolute value equations and inequalities. graph and solve linear and absolute value inequalities in two variables. solve linear systems by graphing. solve a system of linear inequalities by graphing. find the constraints or limitations of a real-world situation by using systems of inequalities. solve a real-world problem using linear programming. graph complex numbers and find their absolute value. Find the sum of complex numbers graphically. graph quadratic functions using technology. Identify vertex, maximum, minimum and line of symmetry. graph parabolas of the form $y = ax^2$. Determine how “a” affects the graph. graph parabolas by using translations to identify features. solve a quadratic equation graphically and find the real roots. find the distance and midpoint between two points in the coordinate plane. Write equations of circles in standard form. Use completing the square to write equations of circles in standard form in order to find the center and radius. write equations of ellipses in standard form. Graph ellipses with center at origin and center at (h,k). write equations of hyperbolas in standard form. Graph hyperbolas with center at the origin and at (h,k). Graph rectangular hyperbolas of the form $xy = k$. solve quadratic-linear systems graphically and algebraically. solve quadratic-quadratic systems graphically and algebraically.
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		<ul style="list-style-type: none">• graph power functions of the form $y = x^n$. Identify point and line symmetry of the graphs.• graph cubic and quartic functions. Determine the degree, the leading coefficient and the constant term of a polynomial function. Determine the characteristics of an odd-degree or an even-degree polynomial function.• find the maximum and minimum values of polynomial functions using technology. Solve real-world problems involving maximum and minimum values.• solve polynomial equations by graphing with technology.• graph rational functions. Determine the discontinuities. Find the vertical asymptotes and the “holes” if they exist.• graph exponential functions. Solve equations by expressing each term as a power with the same base.
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Unit 5 - Exponents, Radicals, and Logarithms, 5 weeks [top](#)

Standards

Algebraic Reasoning: Patterns and Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technology.

1.1 Students should understand and describe patterns and functional relationships.

Core 1.1a Students should describe relationships and make generalizations about patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Core 1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Core 1.3a Students should manipulate equations, inequalities, and functions to solve problems.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technology.

2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships. .

Core 2.1a Students should extend the understanding of number to include integers, rational numbers, and real numbers.

2.1b Students should interpret and represent large sets of numbers with the aid of technology.

Extended 2.1a Students should extend the understanding of number to include the set of complex numbers.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Core 2.2a Students should develop strategies for computation and estimation using properties of number systems to solve problems.

2.2b Students should solve proportional reasoning problems.

Unit Objectives

Students will be able to:

- judge the reasonableness of the results of symbolic manipulations as related to authentic contexts.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- use tables, graphs, and formulas to model exponential growth and decay.
- solve problems involving financial applications including compound interest, and investments.
- evaluate and interpret the graphs of linear, exponential, and polynomial functions.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?

Focus Questions

- How are rational exponents and nth roots of numbers used?
- How are operations performed on functions and their inverses?
- How are radical equations graphed and solved?
- How are exponential, logarithmic, and logistic growth functions graphed and used?
- How are the definitions and properties of

Assessment

Skill Objectives

Students will:

- identify the subsets of the real numbers. Simplify radical expressions.
- solve problems involving direct, inverse, and joint variation.
- graph power functions of the form $y = x^n$. Identify point and line symmetry of the graphs.
- use properties of exponents to simplify expressions.
- convert radical expressions to exponential expressions and vice versa.

	<p>logarithms and the number e used?</p> <ul style="list-style-type: none"> • How are exponential and logarithmic equations solved? 	<ul style="list-style-type: none"> • model data with real number exponents. • solve radical equations algebraically and using technology. Solve literal equations. • graph exponential functions. Solve equations by expressing each term as a power with the same base. • use exponential functions to model exponential growth and decay. • use exponential regression to model real world data. • use the formulas for compound interest and continuous compounding to find the value of an investment. • use the definition of logarithmic functions to convert between logarithmic and exponential form and vice versa. • solve exponential equations using logarithms. • use the properties of logarithms to simplify or expand expressions and to solve equations. • use the change of base formula to evaluate logarithms. • estimate an unknown value between data points on a graph (interpolation) and make predictions by extending the graph (extrapolation).
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Unit 6 – Polynomials, 5 weeks [top](#)

Standards

Algebraic Reasoning: Patterns and Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technology.

1.1 Students should understand and describe patterns and functional relationships.

Core 1.1a Students should describe relationships and make generalizations about patterns and functions.

Extended 1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Core 1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Core 1.3a Students should manipulate equations, inequalities, and functions to solve problems.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technology.

2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships.

Core 2.1a Students should extend the understanding of number to include integers, rational numbers, and real numbers.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Core 2.2a Students should develop strategies for computation and estimation using properties of number systems to solve problems.

Unit Objectives

Students will be able to:

- evaluate and interpret the graphs of linear, exponential, and polynomial functions.
- judge the reasonableness of the results of symbolic manipulations as related to authentic contexts.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- factoring, graphing, and solving polynomials

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?

Focus Questions

- How are operations performed on polynomials?
- How are polynomial equations evaluated, graphed, and solved?

Assessment

- The Painted Cube

Skill Objectives

Students will:

- graph cubic and quartic functions. Determine the degree, the leading coefficient and the constant term of a polynomial function. Determine the characteristics of an odd-degree or an even-degree polynomial function.
- find the maximum and minimum values of polynomial functions using technology. Solve real-world problems involving maximum and minimum values.
- find the real zeros of a polynomial

		<p>function.</p> <ul style="list-style-type: none">• solve polynomial equations by graphing with technology.• find real and complex roots of polynomial equations by dividing and factoring polynomials. Use synthetic division as a tool to divide polynomials. Use the Remainder and Factor Theorems to find factors of a polynomial.• use the rational root theorem to solve polynomial equations.
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