## Mathematics

## Fairfield Public Schools

## GEOMETRY 22



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The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas, organized into six units are as follows.

Critical Area 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems-using a variety of formats-and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations done in the first course. They are able to distinguish whether three given measures (angles or sides) define $0,1,2$, or infinitely many triangles.

Critical Area 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

Critical Area 4: Building on their work with the Pythagorean theorem in 8ın grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

Critical Area 5: In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.
 compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions

| Pacing Guide |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1st Marking Period |  | 2nd Marking Period |  | 3rd Marking Period |  | 4th Marking Period |  |
| September | October No | December | January | February | March | May | June |
| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 |
| Modeling with Geometry, Definitions, and Rigid Motions | Geometric Relationships and Properties | Similarity | Coordinate Geometry | Circles and Conics | Trigonometric Ratios | Geometric Measurement and Dimension | Capstone Project |
| 6 weeks | 5 weeks | 4 weeks | 3 weeks | 3 weeks | 2 weeks | 3 weeks | 2 weeks |


| Course Overview |  |  |
| :---: | :---: | :---: |
| Central Understandings <br> Insights learned from exploring generalizations through the essential questions. (Students should understand that...) <br> - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies. <br> - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools, and technologies. <br> - Shapes and structures can be analyzed, visualized, measured, and transformed using a variety of strategies, tools, and technologies. <br> - Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies. | Essential Questions <br> - How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? <br> - How are quantitative relationships represented by numbers? <br> - How do geometric relationships and measurements help us to solve problems and make sense of our world? <br> - How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decision? | Assessments <br> Individual and district-wide Formative Assessments, Summative Assessments, and Projects. |
| Geometry 22 | BOE Approved **** | 3 |


| Content Outline |  | Standards |
| ---: | :--- | :--- |
| I. | $\frac{\text { Unit } 1 \text { - Modeling with Geometry, Definitions, and }}{\text { Rigid Motions }}$ | CT Common Core State Standards (CTSDE) |
| II. | Unit 2 - Geometric Relationships and Properties |  |
| III. | Unit 3 - Similarity |  |
| IV. | Unit 4 - Coordinate Geometry |  |
| V. | Unit 5 - Circles and Conics |  |
| VI. | Unit 6- Trigonometric Ratios |  |
| VII. | Unit 7 - Geometric Measurement and Dimension |  |
| VIII. | Unit 8 - Capstone Project |  |

## Standards for Mathematical Practice

The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level.

| Standards | Explanations and Examples |
| :--- | :--- | \left\lvert\, \(\left.\begin{array}{ll}Students are expected to: <br>

1. Make sense of problems <br>
and persevere in solving <br>
them.\end{array} \quad $$
\begin{array}{l}\text { High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its } \\
\text { solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution } \\
\text { and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases } \\
\text { and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change } \\
\text { course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the } \\
\text { viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences } \\
\text { between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and } \\
\text { search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this } \\
\text { make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different } \\
\text { approaches. }\end{array}
$$\right.\right\}\)

| Students are expected to: <br> 6. Attend to precision. | By high school, students look closely to discern a pattern or structure. In the expression $x^{2}+9 x+14$, older students can see the 14 as $2 \times$ 7 and the 9 as $2+7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y) 2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures. |
| :---: | :---: |
| Students are expected to: <br> 7. Look for and make use of structure. | High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1),(x-1)\left(x^{2}+x+1\right)$, and $(x-1)\left(x^{3}+x^{2}+x+1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. |
| Students are expected to: 8. Look for and express regularity in repeated reasoning. | By high school, students look closely to discern a pattern or structure. In the expression $x^{2}+9 x+14$, older students can see the 14 as $2 \times$ 7 and the 9 as $2+7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y)^{2}$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures. |

## Unit 1 - Modeling with Geometry, Definitions, and Rigid Motions, 6 weeks top

Students explore geometric ideas using key tools of geometry. This unit focuses on classical Euclidean compass and straightedge constructions, both by hand and in dynamic geometry environments. The goal is to both create constructions and explore why they work. The unit then considers other tools, such as string, paper folding, etc. As with the compass and straightedge constructions, it is important to both mechanically use these tools and to work toward mathematical explanations and justifications. In dynamic geometry environments, a key distinction is between drawing and constructing figure with particular characteristics. For example, if a rectangle is merely drawn to look like a rectangle, then it is easy to mess up the figure by dragging parts of the diagram. But a rectangle that is "constructed" as a rectangle will remain rectangular even when parts of the figure are dragged about.
Building upon the informal experiences with basic geometric objects and relationships, the goal is to start increasing the precision of the definitions. The emphasis should be on the role of definitions and communicating mathematical explanations and arguments rather than on developing a deductive axiomatic system. In Grade 8 rigid motions were explored, here they are defined more precisely and their properties explored.

## Big Ideas

- Understanding the language of Geometry is essential to reasoning and making connections.
- Transformations assist students in developing knowledge of congruence.
- Working with a variety of tools enhances understanding of geometric relationships.


## Essential Questions

- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How does geometry model the physical world?
- How can the language of geometry be used to communicate mathematical ideas coherently and precisely?
- How does the language of geometry provide immediate experience with the physical world?
- How do transformations provide a way of studying figures?

How does the geometric principle of congruence in triangles apply to the real world?

## Standards

## Congruence $G$-CO

Experiment with transformations in the plane.

## G-CO 1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, and plane, and distance along a line (length of a segment), G-CO 3
Given of a rectangle, parallelogram, trapezoid, or regular polygon, and describe the rotations and reflections that carry it onto itself.

## Make geometric constructions.

## G-CO 12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment and constructing a line parallel to a given line through a point not on the line.

## G- CO 13

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Experiment with transformations in the plane.

## G-CO 2

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Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

## G-CO 4

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

## G-CO 5

Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

## G-CO 6

Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

## G-CO 7

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

## G-CO 8

Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

## Unit 2: Geometric Relationships and Properties, 5 weeks top

This unit brings together many of the "classic" theorems of geometry. The emphasis should be on the many roles of proof (a la Devilliers) and a focus on the mathematical practice of making viable arguments and critiquing the reasoning of others. Multiple representations (coordinates, sythemetic, and algebraic proofs of properties can be analyzed and compared).

## Big Ideas

- Geometric ideas are supported and derived using theorems, postulates, definitions, and properties.
- Formal proofs are created using a variety of formats.

Essential Questions

- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How do reasoning and proofs provide the ideas and concepts that lead to an understanding of the deductive nature of geometry?
- How can various types of reasoning be used to make, investigate, and prove mathematical conjectures?
- How do parallel lines, transversals, and related angles model the physical world?
- How does the geometric principle of congruence in triangles apply to the real world?
- How do triangles, their sides, angles, and special segments model the physical world?
- How does the set of quadrilaterals and their properties model the world around us?


## Standards

Congruence G-CO
Prove geometric theorems.
G-CO-Fairfield 1
Develop an understanding of conditionals, converses, and definitions and how they are used in proofs.

## G-CO 9

Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

## G-CO 10

Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to $180^{\circ}$; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

## G-CO 11

Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

## Unit 3 -Similarity, 4 weeks top

Similarity is defined using congruence and dilation. This geometric transformation definition of similarity and congruence is more general than comparing sides and angles. Define a similarity transformation as the composition of a dilation followed by a congruence and prove that the meaning of similarity for triangles is the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. The connection is made between similarity and linearity (why the graph of a linear function is a straight line).

This formalizes the Grade 8 standard 8.EE 6: Understand the connections between proportional relationships, lines, and linear equations.

Big Ideas

- Dilations and similarity are interconnected.
- Properties of similar figures are used to solve problems.
- Formally prove geometric relationships related to similarity.
- Use similarity to measure indirectly.

Essential Questions

- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How does geometry model the physical world?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How are appropriate techniques, tools, and formulas used in geometry to determine measurements?
- How can a variety of appropriate strategies be applied in solving geometric problems?
- How is similarity used to measure indirectly and explore comparable objects?


## Standards

Similarity, Right Triangles, and Trigonometry G-SRT
Understand similarity in terms of similarity transformations.

## G-SRT 1

Verify experimentally the properties of dilations given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

## G-SRT 2

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Explore properties of similarity and prove theorems involving similarity.

## G-SRT 3

Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

## G-SRT 4

Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean
Theorem proved using triangle similarity.

## G-SRT 5

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

## Unit 4 - Coordinate Geometry, 3 weeks top

Making connections between geometric relationships and properties and algebraic expressions is a powerful mathematical tool. This unit provides opportunities to go back and forth between the algebra and geometry. It also builds upon the previous unit making connections between similarity and linearity, for example in $f(x)=m x$ the slope $m$ can be seen as a shape invariant of a right triangle and the $x$ can viewed as the scale factor of the dilation centered at the origin.

Coordinates were introduced in grade 5:
Grade 5 Geometry 1: Graph points on the coordinate plane to solve real-world and mathematical problems.

## Big Ideas

- Integrate algebraic skills and formulas to solve geometry problems.
- Use algebraic skills to prove various geometric properties.


## Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How does geometry model the physical world?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How does algebra relate to geometry graphically?


## Standards

Expressing Geometric Properties with Equations G-GPE
Use coordinates to prove simple geometric theorems algebraically.

## G-GPE 4

Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{ } 3)$ lies on the circle centered at the origin and containing the point $(0,2)$.

## G-GPE 5

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

## G-GPE 6

Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

## G-GPE 7

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

* Denotes a modeling standard in the CCSS


## Unit 5 - Circles and Conics, 3 weeks top

This unit includes key results about circles, treated both synthetically, with attention to constructions, and analytically. Conic sections can arise out of mechanical constructions (e.g. loop of string around two fixed nails to produce an ellipse, paper folding to produce a parabola). Applications can be considered such as elliptic orbits, parabolic motion of projectiles.

## Big Ideas

- Identify various relationships among angles, arcs, and segments in a circle.
- Derive equations and relationships for circles and parabolas.


## Essential Questions

- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?
- How does geometry model the physical world?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How can a variety of appropriate strategies be applied in solving geometric problems?
- How do circles and their parts relate to the physical world?


## Standards

Circles G-C
Understand and apply theorems about circles.

## G-C 1

Prove that all circles are similar.

## G-C 2

Identify and describe relationships among inscribed angles, radii, and chords, and find the distance around a circular arc. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

## G-C 5

Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

## G-C 3

Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
Expressing Geometric Properties with Equations G-GPE
Translate between the geometric description and the equation for a conic section.

## G-GPE 1

Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

## G-GPE 2

Derive the equation of a parabola given a focus and directrix.
(+) denotes a standard in Geometry 21 and not in Geometry 22

Geometry 22

## Unit 6 - Trigonometric Ratios, 2 weeks top

The class of right triangles can be classified up to similarity using "within" comparisons, i.e. the ratio of the lengths of two edges in a right triangle is an important "shape invariant". These ratios have been given names: sine = opposite/hypotenuse, cosine = adjacent/hypotenuse and tangent = opposite/adjacent.

## Big Ideas

- Understand and apply right triangle trigonometry to solve a variety of problems.
- Solve real world problems using right triangle trigonometry.
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$\square$

How can a variety of approp geometric problems?

- How is trigonometry used to understand the functional and aesthetic uses of right triangles?
- How do triangles, their sides, angles, and special segments model the physical world?


## Standards

Similarity, Right Triangles, and Trigonometry G-SRT
Define trigonometric ratios and solve problems involving right triangles.

## G-SRT 6

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G-SRT 7
Explain and use the relationship between the sine and cosine of complementary angles.

## G-SRT 8

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*
G-SRT - Fairfield 1
Find missing sides of special right triangles (30-60-90 \& 45-45-90) given two sides.
G-SRT - Fairfield 2
Find the area of regular polygons

* denotes a modeling standard in the CCSS


## Unit 7 - Geometric Measurement and Dimension, 3 weeks top

This unit should treat geometric measurement more comprehensively and make connections and explain the various volume formulae with attention to the structure of the algebraic expressions. Moving between 2 and 3 dimensions such as via slicing and rotation, and establishing phenomena as fundamental contexts for quadratic relationships and volume phenomena as fundamental contexts for cubic relationships.
This builds on the 7th grade geometry standard: 7G-G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

## Big Ideas

- Use two-dimensional relationships to develop an understanding of threedimensional figures.
- Use algebraic skills in conjunction with previous geometric knowledge to calculate surface area and volume of three-dimensional figures.
- Solve problems using one-two- and three-dimensional ratios.


## Essential Questions

- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How does geometry model the physical world?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How can a variety of appropriate strategies be applied in solving geometric problems?
- How do the calculations and concepts of area and volume relate to two and three-dimensional objects?


## Standards

Geometric Measurement and Dimension G-GMD
Explain volume formulas and use them to solve problems.

## G-GMD 1

Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a prism, cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
G-GMD 3
Use volume formulas for (prisms), cylinders, pyramids, cones, and spheres to solve problems.
G-GMD-Fairfield 1
Calculate the missing side, surface area, and volume using linear, area, and volume ratios respectively.
Geometric Measurement and Dimension G-GMD
Visualize relationships between two-dimensional and three-dimensional objects.
G-GMD 4
Identify the shapes of two-dimensional cross-sections of three- dimensional objects, and identify three-dimensional objects generated by rotations of twodimensional objects.

## Unit 8 - Capstone Geometric Modeling Project, 2 weeks top

The purpose of this unit is to provide closure to the year by wrapping up the big ideas that students learned and synthesize a project.

| Big Ideas |
| :--- | :--- |
| $\bullet \quad$Geometry and other mathematical ideas allow individuals to solve open-ended <br> and complex problems. |

Essential Questions
and complex problems.
Standards
Modeling with Geometry G-MG
Apply geometric concepts in modeling situations.
G-MG 1
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* G-MG 2
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

## G-MG 3

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

* denotes a modeling standard in the CCSS

