INTRODUCTION TO CALCULUS 50

Description

Introduction to Calculus designed for the student who has completed precalculus and wishes to be introduced to a college calculus experience. Topics include a study of relations, functions and their graphs, limits, continuity, differentiation, integration, and applications of these concepts. A major portion of the course will also be devoted to how and why these concepts can be applied in the solving of problems.

Beginning in the 2007-2008 school year, by virtue of our affiliation with the University of Connecticut's Early College Experience (ECE) program, students can apply for 4 college credits of Math 112Q at the University of Connecticut.

Course Overview

Course Goals

Students should:

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

Assessments

Common Assessments

Skill Assessments

Content Outline

I. Unit 1 - Functions, Graphs, and Limits
II. Unit 2 - Differentiation
III. Unit 3 - Applications of the Derivative
IV. Unit 4 - Exponential and Logarithmic Functions
V. Unit 5 - Integration and its Applications

Standards

State of Connecticut Mathematics Curriculum Frameworks

Connecticut State Standards are met in the following areas:

- ***Algebraic Reasoning: Patterns And Functions***
- ***Numerical and Proportional Reasoning***
- ***Geometry and Measurement***
- ***Working with Data: Probability and Statistics***

Grade Level Skills

Students will:

- Skills Matrix

Pacing Guide										
1st Marking Period			2nd Marking Period			3rd Marking Period			4th Marking Period	
September	October	November	December	January	February	March	April	May	June	
Unit 1 **Functions, Graphs, and Limits** 4 weeks	Unit 2 **Differentiation** 5 weeks		Unit 3 **Applications of the Derivative** 8 weeks		Unit 4 **Exponential and Logarithmic Functions** 3 weeks	Unit 5 **Integration and its Applications** 11 weeks				

Unit 1 - Functions, Graphs, and Limits, 4 weeks top

Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

1.1 Students should understand and describe patterns and functional relationships.

Extended 1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Extended 1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Extended 1.3a Students will use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

Extended 3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Working with Data: Probability and Statistics – Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.

4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.

Extended 4.1a Students will model real data graphically using appropriate tools, technology and strategies.

Unit Objectives

Students will be able to:

- find the limit of a function and discuss the continuity of a variety of functions.
- define, explain and graph the limit of a function at a point.
- define, explain and graph a function that is continuous or determine discontinuity.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

Focus Questions

Assessment

- A Cleaner Fairfield

Skill Objectives

Students will:

- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or

	• What is calculus and what role does it play as a tool in science, business, and other areas of study? • How do we analyze various functions to be able to discuss and evaluate their limits and continuities/discontinuities?	relations. • use Cartesian systems to represent, analyze, and solve geometric and measurement problems. • use upper and lower bounds and limits to solve measurement problems. • investigate and solve relevant problems, through designing statistical experiments and collecting, organizing, displaying, and analyzing data in tabular, graphical, and symbolic forms. • recognize the limitations of mathematical models based on sample data as representations of real-world situations. • solve problems involving financial applications including compound interest. • analyze essential relations in a problem to determine possible functions that could model the situation. • understand the definition of and apply the concepts/properties of limits to functions. • determine a limit using graphs, direct substitution and the cancellation technique. • understand the definition of continuity, determine whether a function is continuous on a closed interval, and distinguish between removable and non-removable discontinuities. • combine, compose, and invert functions.

Unit 2 – Differentiation, 5 weeks top

Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

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Extended 1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Extended 1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

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Working with Data: Probability and Statistics – Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.

4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.

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Unit Objectives

Students will be able to:

- determine the slope of a graph and calculate derivatives using the limit definition.
- use the power, product, quotient, and chain rules to determine the derivative.
- apply derivatives to determine rates of change.
- calculate higher order derivatives and implicit differentiation.
- define, explain, evaluate and graph the derivative of a function.
- use derivatives to determine equations of

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

Focus Questions

- What is calculus and what role does it play as a tool in science, business, and other areas of

Assessment

- Cost of Concert Tickets

Skill Objectives

Students will:

- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- use Cartesian systems to represent,

tangents to a curve, evaluate velocity and acceleration of objects, and analyze the graphs of functions.	study? • What is a derivative, how do we determine it, and what are its applications in the real world? • Given data that satisfies a function, how can its derivative and/or integral be found or approximated?	analyze, and solve geometric and measurement problems. • use upper and lower bounds and limits to solve measurement problems. • investigate and solve relevant problems, through designing statistical experiments and collecting, organizing, displaying, and analyzing data in tabular, graphical, and symbolic forms. • recognize the limitations of mathematical models based on sample data as representations of real-world situations. • solve problems involving financial applications including compound interest. • analyze essential relations in a problem to determine possible functions that could model the situation. • understand the definition of and apply the concepts/properties of limits to functions. • determine a limit using graphs, direct substitution and the cancellation technique. • understand the definition of continuity, determine whether a function is continuous on a closed interval, and distinguish between removable and non-removable discontinuities. • combine, compose, and invert functions.

Unit 3 - Applications of the Derivative, 8 weeks top

Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

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Extended 1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

Extended 3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Working with Data: Probability and Statistics – Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.

4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.

Extended 4.1a Students will model real data graphically using appropriate tools, technology and strategies.

Unit Objectives

Students will be able to:

- determine open intervals in which a function is increasing or decreasing.
- determine relative and absolute extrema of a function.
- determine concavity and points of inflection of a graph.
- solve real life optimization problems.
- use calculus to analyze the shape of a function.
- use derivatives to determine equations of tangents to a curve, evaluate velocity and acceleration of objects, and analyze the

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?

Focus Questions

- What is calculus and what role does it

Assessment

- Coffee Demand

Skill Objectives

Students will:

- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- use Cartesian systems to represent, analyze, and solve geometric and measurement problems.

graphs of functions.	play as a tool in science, business, and other areas of study? • How can calculus and the concepts of limit and continuity assist us in analyzing curves? • What is a derivative, how do we determine it, and what are its applications in the real world? • Given data that satisfies a function, how can its derivative and/or integral be found or approximated?	• use upper and lower bounds and limits to solve measurement problems. • investigate and solve relevant problems, through designing statistical experiments and collecting, organizing, displaying, and analyzing data in tabular, graphical, and symbolic forms. • recognize the limitations of mathematical models based on sample data as representations of real-world situations. • determine and use critical numbers to establish increasing and decreasing intervals. • understand and use the first derivatives test and the extreme value theorem to determine the relative and absolute extrema. • understand and use second derivatives to determine the concavity and points of inflections of a function over a given interval. • understand the definition of asymptotes and how vertical and horizontal asymptotes are related to infinite limits. • use continuity, derivatives, extrema, concavity, points of inflection and asymptotes to sketch a curve. • analyze essential relations in a problem to determine possible functions that could model the situation. • solve problems involving financial applications including compound interest. • understand and use optimization strategies. • recognize that the slope of the tangent line to a curve represents the rate of change. • apply the concepts of limits to sequences and asymptotic behavior of functions.

Unit 4 - Exponential and Logarithmic Functions, 3 weeks top

Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

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Extended 1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Extended 1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

Extended 1.3a Students will use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Extended 2.2a Students will investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

Working with Data: Probability and Statistics – Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.

4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.

Extended 4.1a Students will model real data graphically using appropriate tools, technology and strategies.

Unit Objectives

Students will be able to:

- determine the derivative of exponential and logarithmic functions.
- use calculus to solve exponential growth and decay problems.
- define, explain and graph exponential and logarithmic functions and their inverse relationship.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying

Assessment

- Learning Curve

Skill Objectives

Students will:

- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.

	data help us analyze information and make reasonable predictions and informed decisions? **Focus Questions** • What is calculus and what role does it play as a tool in science, business, and other areas of study? • Given data that satisfies a function, how can its derivative and/or integral be found or approximated?	• recognize the effect of changes in parameters on the graphs of functions or relations. • use Cartesian systems to represent, analyze, and solve geometric and measurement problems. • use upper and lower bounds and limits to solve measurement problems. • investigate and solve relevant problems, through designing statistical experiments and collecting, organizing, displaying, and analyzing data in tabular, graphical, and symbolic forms. • recognize the limitations of mathematical models based on sample data as representations of real-world situations. • apply the concepts of limits to sequences and asymptotic behavior of functions. • create, apply and solve models of exponential growth and decay and use their derivatives to solve real-world situations. • apply inverse properties to logarithmic and exponential functions. • describe and compare properties, graphs and classes of functions including exponential and logarithmic. • solve problems involving financial applications including compound interest. • recognize that the slope of the tangent line to a curve represents the rate of change. • use logarithms to solve problems. • perform operations with logarithms including the derivative.

Unit 5 - Integration and its Applications, 11 weeks top

Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

1.1 Students should understand and describe patterns and functional relationships.

Extended 1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Extended 1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Extended 2.2a Students will investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

Extended 3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Working with Data: Probability and Statistics – Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.

4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.

Extended 4.1a Students will model real data graphically using appropriate tools, technology and strategies.

Unit Objectives

Students will be able to:

- define, explain and evaluate definite and indefinite integrals.
- determine the antiderivative of a function.
- use the general power rule, exponential and log rule to calculate antiderivative.
- evaluate definite integrals and apply the Fundamental Theorem of Calculus to

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Assessment

- Where's The Beef?

Skill Objectives

Students will:

- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of

determine the area bounded by two graphs.	• How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? **Focus Questions** • What is calculus and what role does it play as a tool in science, business, and other areas of study? • What is an integral (definite and indefinite), how can it be determined and/or evaluated and how can it be applied to problems in the real world? • Given data that satisfies a function, how can its derivative and/or integral be found or approximated?	symmetry for that function. • recognize the effect of changes in parameters on the graphs of functions or relations. • use Cartesian systems to represent, analyze, and solve geometric and measurement problems. • use upper and lower bounds and limits to solve measurement problems. • investigate and solve relevant problems, through designing statistical experiments and collecting, organizing, displaying, and analyzing data in tabular, graphical, and symbolic forms. • recognize the limitations of mathematical models based on sample data as representations of real-world situations. • understand and determine antiderivatives. • determine an indefinite integral and a particular solution to an integral. • use above to apply to problems such as position functions and cost functions. • understand and apply the general power rule. • determine the indefinite and definite integral of an exponential and logarithmic function. • use the Fundamental Theorem of Calculus to determine the area between two graphs. • understand and use the average of a function in problem solving. • perform operations with logarithms including the derivative.