MULTIVARIABLE CALCULUS 61

Description

Multivariable Calculus is a rigorous second year course in college level calculus. This course provides an in-depth study of vectors and the calculus of several variables for the student who has successfully completed Calculus BC. The successful student will bring to the course a solid understanding of the concepts of first-year calculus as well as the ability to approach complex problems and applications with insight, imagination, and persistence. Major topics will include vector operations and analysis, functions of two or more variables and their partial derivatives, multiple integration.

Course Overview

Course Goals

Students should:

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Assessments

Common Assessments

Skill Assessments

Content Outline

I. Unit 1 - Vectors and the Geometry of Space
II. Unit 2 - Vector Functions
III. Unit 3 - Partial Derivatives
IV. Unit 4 - Multiple Integrals

Standards

State of Connecticut Mathematics Curriculum Frameworks

Connecticut State Standards are met in the following areas:

- ***Algebraic Reasoning: Patterns And Functions***
- ***Numerical and Proportional Reasoning***
- ***Geometry and Measurement***

Grade Level Skills

Students will:

- Skills Matrix

Pacing Guide			
1st Marking Period	2nd Marking Period	3rd Marking Period	4th Marking Period
September October November December January February March April May June			
Unit 1 **Vectors and the Geometry of Space** 10 weeks	Unit 2 **Vector Functions** 8 weeks	Unit 3 **Partial Derivatives** 10 weeks	Unit 4 Multiple Integrals 10 weeks

Unit 1 - Vectors and the Geometry of Space, 10 weeks top

Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

1.1 Students should understand and describe patterns and functional relationships.

Extended 1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Extended 1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

Extended 2.2a Students should investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

Unit Objectives

Students will be able to:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- recognize the effect of changes in parameters on the graphs of multivariable

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is a vector?
- What operations can be performed using vectors?

Assessment

- Hanging a Sign

Skill Objectives

Students will:

- graph points in planes in three dimensions.
- use the distance formula.
- describe and sketch regions involving spheres.
- describe vectors algebraically, graphically, and verbally.
- compute with vectors using properties of vectors and vector operations.
- solve problems involving force and

functions or relations. • recognize vectors as systems that have some, but not all, of the properties of real numbers.	• How can you use vectors to determine equations of lines and planes in space?	velocity using components of vectors. • compute dot product of vectors. • use dot product to determine the angle between vectors. • find scalar and vector projections. • compute and use cross products. • compute and use scalar and vector triple products. • compute and use vector and parametric equations of lines. • compute and use equations of planes. • sketch graphs and describe properties of functions in two variables. • work with quadric surfaces. • convert from one coordinate system to another. • graph functions given in spherical and cylindrical coordinates.

Unit 2 – Vector Functions, 8 weeks top

Standards

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Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

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Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

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Unit Objectives

Students will be able to:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- relate the graphical representation of a function to its function family and find

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is a vector function?
- What is a derivative/integral of a vector function?

Assessment

- Fireworks Display

Skill Objectives

Students will:

- sketch and recognize graphs of space curves given parametrically.
- find the domain, range and limits of vector functions.
- compute derivatives of vector functions.
- determine tangent vectors of space curves.
- compute integrals of vector functions.
- compute arc length of space curves.

equations, intercepts, maximum or minimum values, asymptotes and symmetries for multivariable functions. • recognize vectors as systems that have some, but not all, of the properties of real numbers.	• What is curvature? • What is a parametric surface?	• parameterize curves with respect to arc length. • compute the curvature of a curve. • compute the normal and binormal vectors and the associated planes of a curve. • compute the velocity and acceleration of a particle. • solve problems involving motion, acceleration or force. • graph and recognize parametric surfaces. • find parametric representations of surfaces.

Unit 3 - Partial Derivatives, 10 weeks top

Standards

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Extended 1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Extended 1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

Extended 3.3a Students should approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Unit Objectives

Students will be able to:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- approximate methods to solve measurement problems using multivariable functions.
- understand and use optimization strategies including maximums and minimums.
- apply the concepts of limits and asymptotic behavior of multivariable

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is a partial derivative?
- What is a directional derivative?
- What is a gradient vector?
- What is a LaGrange Multiplier?

Assessment

- Wind Chill Effect

Skill Objectives

Students will:

- find domain and range of functions of two variables.
- describe the level surfaces of functions with three variables.
- determine if a function f(x,y), has a limit at (a,b).
- determine the points of continuity of a function f(x,y).
- compute and interpret partial derivatives.
- use Clairault's Theorem to compute higher partial derivatives.

functions. • relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and symmetries for multivariable functions. • recognize that the slope of the tangent line to the surface represents the rate of change in a particular direction. • visualize three-dimensional objects from different perspectives and analyze cross-section and volume.		• verify whether or not a given function satisfies a partial differential equation. • compute the tangent plane to a surface given by a function of two variables. • determine if a function is differentiable. • use linearization to approximate values of a function. • compute tangent planes to parametric surfaces. • compute derivatives using the Chain Rule. • use implicit differentiation to compute derivatives. • compute directional derivatives. • find an apply the gradient vector. • find tangent planes and normal lines to level surfaces. • find local maximum and minimum values. • compute absolute maximum and minimum values of a function. • determine the saddle points of a function, if any. • use the method of Lagrange multipliers to determine extreme values of a function subject to constraints.

Unit 4 - Multiple Integrals, 10 weeks top

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1.2 Students should represent and analyze quantitative relationships in a variety of ways.

Extended 1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

Extended 3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

Extended 3.3a Students should approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Unit Objectives

Students will be able to:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- recognize the effect of changes in parameters on the graphs of multivariable functions or relations.
- examine the effects of transformations on

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
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Focus Questions

- What is an iterated integral?
- What is a Jacobian?

Assessment

- Swimming Pool

Skill Objectives

Students will:

- use a double Reimann Sum to approximate double integrals.
- evaluate double integrals by computing volumes.
- evaluate double integrals over general regions.
- evaluate double integrals over polar regions.
- use double integrals to compute mass.
- compute moments of inertia and centers of mass.

multivariable functions. • approximate methods to solve measurement problems using multivariable functions.		• compute triple integrals. • apply Fubini's Theorem. • apply triple integrals to problems of volume, density and mass. • compute triple integrals using cylindrical coordinates. • compute triple integrals using spherical coordinates. • find the image of a set under a transformation. • compute the Jacobian of a transformation. • use change of variables to simplify and evaluate multiple integrals.