# Mathematics

# Fairfield Public Schools

## Pre-Algebra 7

# PRE-ALGEBRA 7

## Critical Areas of Focus

In the Bridge to Algebra Course, instructional time should focus on five critical areas: (1) developing understanding of and applying proportional relationships; (2) drawing inferences about populations based on samples, (3) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (4) grasping the concept of a function and using functions to describe quantitative relationships; (5) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.
2. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.
3. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality ($m$) is the slope, and the graphs are lines through the origin. They understand that the slope ($m$) of a line is a constant rate of change, so that if the input or $x$-coordinate changes by an amount $A$, the output or $y$-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and $y$-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.
4. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.
5. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

## Pacing Guide

| 1st Marking Period | 2nd Marking Period | 3rd Marking Period | 4th Marking Period |
|---|---|---|---|

| September | October | November | December | January | February | March | April | May | June |
|---|---|---|---|---|---|---|---|---|---|

| Unit 1 | Unit 2 | Unit 3 | Unit 4 | Unit 5 | Unit 6 | Unit 7 | Unit 8 | Unit 9 |
|---|---|---|---|---|---|---|---|---|
| **Proportional Relationships** | **Algebraic Reasoning II** | **Inferences about Populations** | **Probability** | **Pythagorean Theorem** | **Congruence and Similarity** | **Linear Relationships** | **Systems of Linear Relationships** | **Volume** |
| 5 weeks | 4 weeks | 3 weeks | 3 weeks | 4 weeks | 3 weeks | 5 weeks | 3 weeks | 3 weeks |



## Course Overview

**Central Understandings**

Insights learned from exploring generalizations through the essential questions. (Students will understand that…)

- Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies.
- Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
- Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools, and technologies.
- Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.

**Essential Questions**

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?

**Assessments**

- Formative Assessments
- Summative Assessments

| **Content Outline** | **Standards** |
|---|---|
| I. Unit 1 – Proportional Relationships<br>II. Unit 2 – Algebraic Reasoning II<br>III. Unit 3 – Inferences about Populations<br>IV. Unit 4 – Probability<br>V. Unit 5 - Pythagorean Theorem<br>VI. Unit 6 – Congruence and Similarity<br>VII. Unit 7 – Linear Relationships<br>VIII. Unit 8 – Systems of Linear Relationships<br>IX. Unit 9 – Volume | Connecticut Common Core State Standards are met in the following areas:<br>• ***Ratios and Proportional Relationships***<br>• ***Expressions and Equations***<br>• ***Geometry***<br>• ***Statistics and Probability***<br>• ***Functions*** |

## Grade Seven Standards for Mathematical Practice

The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

| *Standards* | *Explanations and Examples* |
|---|---|
| **1. Make sense of problems and persevere in solving them**. | In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" |
| **2. Reason abstractly and quantitatively.** | In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations. |
| **3. Construct viable arguments and critique the reasoning of others.** | In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking. |
| **4. Model with mathematics.** | In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context. |
| **5. Use appropriate tools strategically.** | Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms. |
| **6. Attend to precision.** | In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities. |
| **7. Look for and make use of structure.** | Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3(2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality), $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities. |
| **8. Look for and express regularity in repeated reasoning**. | In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events. |

| Unit 1 – Proportional Relationships, 5 weeks | |
|---|---|
| Students studied rates and ratios in sixth grade. In this unit, students reason proportionally. They use ratios as a basis of comparison between two sets of data. They observe related data in the form of a table and look for patterns connecting these data values. Plotting the paired data points to see a graphical representation, and writing an equation that shows the relationship of the data in the table further strengthens this understanding. When the change observed in the table is constant, students connect to a linear graph. This demonstrates a proportional relationship across multiple representations and deepens the understanding of these characteristics. The unit rate studied in grade six is now a focus of rate of change used in writing linear equations in grade seven.<br><br>Other concepts in this unit include solving problems to find an unknown part of a proportion and applying proportional reasoning to real-world contexts. Students think proportionally in such situations as calculating sales tax, interest, and commissions; scale drawings; and unit pricing. | |
| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
| • Reasoning with ratios involves attending to and coordinating two quantities.<br>• Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.<br>• Ratios are often used to make "part-to-part" comparisons, but fractions are not.<br>• Equivalent ratios can be created by iterating and/or partitioning a composed unit.<br>• A rate is a set of infinitely many equivalent ratios.<br>• Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.<br>• A proportion is a relationship of equality between two ratios. | • How do you find and compare unit rates?<br>• How can you use tables and equations to identify and describe proportional relationships?<br>• How can you use graphs to represent and analyze proportional relationships?<br>• How do you use percents to solve problems? |
| **Common Core State Standards** | |
| **RATIOS AND PROPORTIONAL RELATIONSHIPS**<br>**Analyze proportional relationships and use them to solve real-world and mathematical problems.**<br>**7.RP.1**<br>Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ to ¼ miles per hour, equivalently 2 miles per hour.*<br>**7.RP.2**<br>Recognize and represent proportional relationships between quantities.<br>**7.RP.2a**<br>Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.<br>**7.RP.2b**<br>Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. | |

**7.RP.2c**

Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.

**7.RP.2d**

Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where $r$ is the unit rate.

**7.RP.3**

Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error*.

**GEOMETRY**

**Draw, construct, and describe geometrical figures and describe the relationships between them.**

**7.G.1**

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

## Unit 2 – Algebraic Reasoning II, 4 weeks

The main goal of this unit is for students to be able to recognize mathematical characteristics of proportional situations and to be able to identify, represent, and interpret such relationships. To do this, students explore multiple representations (i.e., graphs, words, equations, and tables) to interpret and solve tasks and understand how these models relate to one another.

To develop such understanding, students need to explore situations involving constant rate of change between two variables and be asked to express and represent the situation with a verbal description, a table, a graph, and/or an equation. The unit should involve lessons in which students experience constant rates of change and are asked to identify the rate of change, such as driving a car at a constant rate over a period of time. They also need to explore non-proportional situations where as one variable changes, the other variable continues to change at a constant rate but the relationship between any two points is no longer proportional, such as in a purchasing plan where there is a start-up fee in addition to a constant per item fee. Doing this prior to learning how to formally solve equations can help students make sense of the constant term in an equation and eventually learn to write equivalent equations to solve. Contrasting these different types of situations creates opportunities to deepen students understanding of proportional relationships, and lays the groundwork for introducing slope.

By the end of the unit, students should be able to determine whether relationships are changing at a constant rate, if they are proportional or non-proportional, and strategically and efficiently use a variety of representations to reason about these relationships. By comparing table and graphs, and looking for common solutions/points of intersection and discussing the meaning of these solutions students are informally solving systems of equations. This unit sets the foundation for work in eighth grade using linear functions to model relationships between quantities. In eighth grade students will also formalize their mathematical language (e.g., slope) and rely more on symbolic representations as appropriate.

### Big Ideas

The central organizing ideas and underlying structures of mathematics

- The equal sign indicates that two expressions are equivalent. It can also be used in defining or naming a single expression or function rule.
- An inequality is another way to describe a relationship between expressions; instead of showing that the values of two expressions are equal, inequalities indicate that the value of one expression is greater than (or greater than or equal to) the value of the other expression.

### Essential Questions

- How do you add, subtract, factor, and multiply algebraic expressions?
- How do you solve equations that contain multiple operations?
- How do you solve inequalities that involve one operation?
- How do you solve inequalities that involve multiple operations?

### Common Core State Standards

**EXPRESSIONS AND EQUATIONS**
**Use properties of operations to generate equivalent expressions.**

**7.EE.1**
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

**7.EE.2**
Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are

related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5 percent" is the same as "multiply by 1.05."*

**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**

**7.EE.4**

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

**7.EE.4a**

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p, q, and r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

**7.EE.4b**

Solve word problems leading to equations of the form $px + q > r$ *or* $px + q < r$, where *p, q, and r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make and describe the solutions.*

## Unit 3 – Inferences about Populations, 3 weeks

This unit builds off the learning from grade six where the students began to investigate statistical thinking. As students learned approaches to collect and represent data, the concepts in this unit allow students to apply this knowledge and investigate how statistical thinking applies to populations and samples. The students being able to generalize information from a sample allow them to make judgments and assertions of the entire population.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics. | **Essential Questions** |
|---|---|
| • The information from a sample can be used to make generalizations about a population. | • How can you use a sample to gain information about a population?<br>• How can you use samples to make and compare predictions about populations?<br>• How can you use measures of center and variability to compare two populations? |

### Common Core State Standards

**STATISTICS AND PROBABILITY**

**Use random sampling to draw inferences about a population.**

**7.SP.1**

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

**7.SP.2**

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

**Draw informal comparative inferences about two populations.**

**7.SP.3**

Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

**7.SP.4**

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

| **Unit 4 – Probability, 3 weeks** | |
|---|---|
| The overarching goal of this unit is to learn how to make predictions and decisions using knowledge of probability and expected values. Initially, students need to develop an understanding of how numbers represent the likelihood of an event occurring, with higher numbers representing a higher likelihood up to the maximum of 1. They should have opportunities to calculate likelihoods of simple probabilities, and then run trials to determine experimental probabilities, and compare those to the theoretical probabilities.<br><br>Moving into compound probabilities, students should have the opportunity to use many different strategies for representing all the outcomes of a probability situation, such as lists, tables, area models, and tree diagrams. Students should also have the opportunity to make sense of the idea of expected values, and develop a strategy for calculating expected values. | |
| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
| • The probability of an event's occurrence can be predicted with varying degrees of confidence. | • How can you describe the likelihood of an event?<br>• How can you find the theoretical probability of an event?<br>• How do you find the experimental probability of an event?<br>• How do you find the probability of a compound event?<br>• How can you use simulations to estimate probabilities? |

**Common Core State Standards**

**STATISTICS AND PROBABILITY**

**Investigate chance processes and develop, use, and evaluate probability models.**

**7.SP.5**
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

**7.SP.6**
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

**7.SP.7**
Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

**7.SP.7a**
Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

**7.SP.7b**
Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the*

*approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

**7.SP.8**

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

**7.SP.8a**

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

**7.SP.8b**

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the events.

**7.SP.8c**

Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40 percent of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood*

## Unit 5 – Pythagorean Theorem, 4 weeks

The focus of this unit is the development and understanding of the Pythagorean Theorem. Students begin by looking for distances between two points in a coordinate plane. They develop the idea of square roots by considering squares of specific areas which then have side lengths related to these areas. In looking at square roots, students should be able to recognize perfect squares and use this knowledge to estimate the values of square roots of whole numbers. Because it is not possible to use numbers previously studied for all sizes of squares, it becomes important to develop understanding of a new category of numbers called irrational numbers. Discussion of the other classifications of numbers previously studied (counting numbers, whole numbers, integers, and rational numbers) is appropriate at this time. Students explore the Pythagorean Theorem and its proofs. Students then use this knowledge to solve problems related to the Pythagorean Theorem and the converse of the Pythagorean Theorem.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • The Pythagorean theorem holds for any right triangle and the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. | • What is the Pythagorean Theorem and when does it hold?<br>• How can the Pythagorean Theorem be used to find missing lengths in a triangle? |

### Common Core State Standards

**GEOMETRY**
**Understand and apply the Pythagorean Theorem.**

**8.G.6**
Explain a proof of the Pythagorean Theorem and its converse.

**8.G.7**
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

**8.G.8**
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**EXPRESSIONS AND EQUATIONS**
**Work with radicals and integer exponents.**

**8.EE.2**
Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

**Unit 6 – Congruence and Similarity, 3 weeks**

This unit on transformational geometry begins by looking at symmetry. Students are asked to look at figures and designs and determine the types of symmetry that exist in each case. The three types of symmetry used are reflectional, rotational, and translational symmetry. Students will extend their understanding of these symmetry types by using properties to make their own symmetric designs. Students should have access to measurement tools such as hinged mirrors, MIRAs or transparent reflection tools, protractors or angle rulers, straight edges, and compasses to assist in their analysis and design. In studying ideas of symmetry, students will identify lines of reflection, centers of rotation, angles of rotation, and translation directions and magnitudes. This study of symmetry leads to analyzing and understanding symmetry transformations in which distances between pre-image and image points is maintained. These transformations will be used to determine congruence of shapes. Students will informally prove congruence of figures by giving a sequence of transformations that moves one figure to another. When using sequences of transformations, students will observe whether reversing the order of the transformations will yield the same image.

The connection between the geometry and algebra of transformations will be made by placing figures in a coordinate plane and asking students to write coordinate rules that demonstrate each transformation.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • Geometric thinking involves developing, attending to, and learning how to work with imagery.<br>• Decomposing and rearranging provide a geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one.<br>• Symmetry provides a powerful way of working geometrically. | • What types of symmetry can be found around you?<br>• How can transformations be described algebraically?<br>• How can you use a line to generate the reflection image of a figure over that line?<br>• What are the coordinate rules for reflections over the x-axis and y-axis, dilations centered at the origin, and translations?<br>• Why do these rules make sense geometrically?<br>• How do you determine the angles of rotation for a figure that has rotational symmetry?<br>• What does it mean for figures to be similar and/or congruent? |

**Common Core State Standards**

**GEOMETRY**

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

**8.G.1**

Verify experimentally the properties of rotations, reflections, and translations:

**8.G.1a**

Lines are taken to lines, and line segments to line segments of the same length.

**8.G.1b**

Angles are taken to angles of the same measure.

**8.G.1c**

Parallel lines are taken to parallel lines.

**8.G.2**

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

**8.G.3**

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

**8.G.4**

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

**8.G.5**

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

| Unit 7 – Linear Relationships, 5 weeks | |
|---|---|
| Students generalize their knowledge of proportional situations to linear relationships and connect the constant of proportionality seen in tables and graphs to the rate of change (slope). They also represent linear relationships using tables, equations, and graphs. They write equations in $y = mx + b$ form, and learn how the slope ($m$) portrays the relationship between variables. Scatterplots of collected data are used to determine the relationship between the variables. In the case of a linear-like relationship, a line of best fit is applied. Students will represent linear functions by using spreadsheets and using graphing calculators or computer software. Questions about linear relationships in real-world and mathematical contexts are solved using a variety of methods including tables, graphs, and equations. Linear functions are contrasted with directly proportional patterns of change and inverse relations. | |
| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
| • Functions are a single-valued mappings from one set-the domain of the function-to another- its range.<br>• A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model.<br>• Linear functions are characterized by a constant rate of change. Reasoning about the similarity of "slope" triangles allows deducing that linear functions have a constant rate of change and a formula of the type $f(x) = mx + b$.<br>• Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and tables.<br>• One variable equations are the foundation for Algebra and solving them is rooted in mathematical properties (distributive, addition property of equality, multiplication property of equality). | • What is the relationship that describes a function?<br>• How can functions be used to find the output from a given input?<br>• How can equations, graphs, word descriptions, and tables describe a function?<br>• What is a rate and how does is it related to proportional reasoning?<br>• How can you use mathematical properties to solve one variable equations? |
| **Common Core State Standards** | |

**EXPRESSIONS AND EQUATIONS**

**Understand the connections between proportional relationships, lines, and linear equations.**

**8.EE.5**

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

**8.EE.6**

Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.**.**

**8.EE.7**

Solve linear equations in one variable.

**8.EE.7a**

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where *a* and *b* are different numbers).

**8.EE.7b**

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

**FUNCTIONS**

**Define, evaluate, and compare functions.**

**8.F.1**

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

**8.F.2**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

**8.F.3**

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

**8.F.4**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ($x$, $y$) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

**8.F.5**

Describe qualitatively the functional relationship between two quantities by analyzing a graph, (e.g. where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

## Unit 8 – Systems of Linear Relationships, 3 weeks

The focus of this unit is on developing a conceptual understanding of the connections between algebra and geometry in viewing systems of equations. Students explore situations where multiple solutions are possible and write equations to represent these situations. Graphing technology is used to explore solutions to systems of linear equations.

In order to become proficient in solving systems of equations in a variety of ways, students need to become skilled in converting back and forth between the slope-intercept and standard forms of equations. This will allow students to graph equations (even if not given in slope-intercept form) in order to find the solution(s) to the systems. It also allows students to use the substitution method by first solving for x or y in the equations. The other method explored in this unit is the combining equations method. After practicing and comparing these three methods of solving systems, students should be able to make decisions about methods that will be most feasible for a given system.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • Solving a system equation can be done with tables, graphs and equations.<br>• The different methods to solve a system of equations can be more efficient than others, based on the situation and context. | • How can situations be modeled as a system of linear equations and how to find solutions using all constraints?<br>• What does it mean when the graphs of two functions intersect?<br>• What method is most appropriate solve a system of equations (table, graph, or equation)? |

### Common Core State Standards

**EXPRESSIONS AND EQUATIONS**

**Analyze and solve linear equations and pairs of simultaneous linear equations.**

**8.EE.7**

Solve linear equations in one variable.

**8.EE.7a**

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).

**8.EE.7b**

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

**8.EE.8**

Analyze and solve pairs of simultaneous linear equations.

**8.EE.8a**

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

**8.EE.8b**

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

**8.EE.8c**

Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

**FUNCTIONS**

**Define, evaluate, and compare functions.**

**8.F.2**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

**Use functions to model relationships between quantities.**

**8.F.4**

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ($x$, $y$) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

## Unit 9 – Volume, 3 weeks

In this unit, students build off their understanding of area to build an understanding of volume. Students will extend their understandings of volume learned in 7th grade (cubes, and right prisms), to further understanding of cones, cylinders, and spheres. Knowing the formulas for the volumes of cones, cylinders and spheres and applying them to solve real-world and mathematical problems is also a part of this unit. Within the volume formulas, both square roots and cube roots may arise in problem solving situations.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena. | • How can measurements be used to solve problems?<br>• What measurements are needed to find the volume of a 3-dimensional figure?<br>• How are prisms and cylinders alike and how are they different? |

### Common Core State Standards

**GEOMETRY**

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

**8.G.9**

Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.