## Mathematics

## Fairfield Public Schools

## Pre-Algebra 8



## PRE-ALGEBRA 8

## Critical Areas of Focus

In the Pre-Algebra 8 course, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions $(y / x=m$ or $y=m x)$ as special linear equations $(y=m x+b)$, understanding that the constant of proportionality $(m)$ is the slope, and the graphs are lines through the origin. They understand that the slope $(m)$ of a line is a constant rate of change, so that if the input or $x$-coordinate changes by an amount $A$, the output or $y$ coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and $y$-intercept) in terms of the situation.
Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.
2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.
3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.


## Course Overview

## Central Understandings

Insights learned from exploring generalizations through the essential questions. (Students will understand that...)

- Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies.
- Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
- Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools, and technologies.
- Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.


## Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?

Assessments

- Formative Assessments
- Summative Assessments

| Content Outline | Standards |
| :---: | :---: |
| I. Unit 1-Real Numbers | Connecticut Common Core State Standards are met in the following areas: |
| II. Unit 2 - Pythagorean Theorem | - The Number System |
| III. Unit 3 - Congruence and Similarity | - Expressions and Equations |
| IV. Unit 4-Linear Relationships | - Functions |
| V. Unit 5-Systems of Linear Relationships | - Geometry |
| VI. Unit 6 - Volume <br> VII. Unit 7 - Patterns in Data | - Statistics and Probability |

## Grade Eight Standards for Mathematical Practice

The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

| Standards | Explanations and Examples |
| :---: | :---: |
| 1. Make sense of problems and persevere in solving them. | In grade 8, students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" |
| 2. Reason abstractly and quantitatively. | In grade 8, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations. |
| 3. Construct viable arguments and critique the reasoning of others. | In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking. |
| 4. Model with mathematics. | In grade 8, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students solve systems of linear equations and compare properties of functions provided in different forms. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context. |
| 5. Use appropriate tools strategically. | Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal. |
| 6. Attend to precision. | In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. |
| 7. Look for and make use of structure. | Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity |
| 8. Look for and express regularity in repeated reasoning. | In grade 8, students use repeated reasoning to understand algorithms and make generalizations about patterns. Students use iterative processes to determine more precise rational approximations for irrational numbers. During multiple opportunities to solve and model problems, they notice that the slope of a line and rate of change are the same value. Students flexibly make connections between covariance, rates, and representations showing the relationships between quantities. |

## Unit 1 - Real Numbers, 4 weeks top

The focus of this unit is for students to develop the conceptual understanding with real numbers and the properties the encompass them. Students will learn to differentiate between the two different types of real numbers: (a) rational and (b) irrational numbers. Students will also develop the understanding of the rules of exponents, which will also be extended and built upon when they get into high school (N.RN.1, and N.RN.2).

Big Ideas
The central organizing ideas and underlying structures of mathematics

- Rational numbers can be represented in multiple ways and are useful when examining situations involving numbers that are not whole.


## Essential Questions

- How are rational and irrational numbers different?
- How can we compare and contrast numbers?
- When are exponents used and why are they important?
- Why is it useful for me to know the square root of a number?
- How do I simplify and evaluate algebraic expressions involving integer exponents and square roots?
- In what ways can rational numbers be useful?


## Common Core State Standards

## THE NUMBER SYSTEM

## Know that there are numbers that are not rational, and approximate them by rational numbers.

## 8.NS. 1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

## 8.NS. 2

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\sqrt{ } 2$ ). For example, by truncating the decimal expansion of $\sqrt{ } 2$, show that $\sqrt{ } 2$ is between 1 and 2 , then between 1.4 and 1.5, and explain how to continue on to get better approximations.

## EXPRESSIONS AND EQUATIONS

## Work with radicals and integer exponents.

8.EE. 1

Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^{2} \times 3^{-5}=3^{-3}=1 / 3^{3}=1 / 27$.

## 8.EE. 2

Use square root and cube root symbols to represent solutions to equations of the form $x^{2}=p$ and $x^{3}=p$, where $p$ is a positive rational number.
Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{ } 2$ is irrational.

## 8.EE. 3

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^{8}$ and the population of the world as 7 $\times 10^{9}$, and determine that the world population is more than 20 times larger.

## 8.EE. 4

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

## Unit 2 - Pythagorean Theorem, 5 weeks top

The focus of this unit is the development and understanding of the Pythagorean Theorem. Students begin by looking for distances between two points in a coordinate plane. They develop the idea of square roots by considering squares of specific areas which then have side lengths related to these areas. In looking at square roots, students should be able to recognize perfect squares and use this knowledge to estimate the values of square roots of whole numbers. Because it is not possible to use numbers previously studied for all sizes of squares, it becomes important to develop understanding of a new category of numbers called irrational numbers. Discussion of the other classifications of numbers previously studied (counting numbers, whole numbers, integers, and rational numbers) is appropriate at this time. Students explore the Pythagorean Theorem and its proofs. Students then use this knowledge to solve problems related to the Pythagorean Theorem and the converse of the Pythagorean Theorem.

## Big Ideas

The central organizing ideas and underlying structures of mathematics

- The Pythagorean theorem holds for any right triangle and the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.


## Essential Questions

- What is the Pythagorean Theorem and when does it hold?
- How can the Pythagorean Theorem be used to find missing lengths in a triangle?


## Common Core State Standards

## GEOMETRY

## Understand and apply the Pythagorean Theorem.

8.G. 6

Explain a proof of the Pythagorean Theorem and its converse.
8.G. 7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

## 8.G. 8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

## EXPRESSIONS AND EQUATIONS

## Work with radicals and integer exponents.

8.EE. 2

Use square root and cube root symbols to represent solutions to equations of the form $x^{2}=p$ and $x^{3}=p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{ } 2$ is irrational.

## Unit 3 - Congruence and Similarity, 5 weeks top

This unit on transformational geometry begins by looking at symmetry. Students are asked to look at figures and designs and determine the types of symmetry that exist in each case. The three types of symmetry used are reflectional, rotational, and translational symmetry. Students will extend their understanding of these symmetry types by using properties to make their own symmetric designs. Students should have access to measurement tools such as hinged mirrors, MIRAs or transparent reflection tools, protractors or angle rulers, straight edges, and compasses to assist in their analysis and design. In studying ideas of symmetry, students will identify lines of reflection, centers of rotation, angles of rotation, and translation directions and magnitudes. This study of symmetry leads to analyzing and understanding symmetry transformations in which distances between pre-image and image points is maintained. These transformations will be used to determine congruence of shapes. Students will informally prove congruence of figures by giving a sequence of transformations that moves one figure to another. When using sequences of transformations, students will observe whether reversing the order of the transformations will yield the same image.

The connection between the geometry and algebra of transformations will be made by placing figures in a coordinate plane and asking students to write coordinate rules that demonstrate each transformation.

## Big Ideas

The central organizing ideas and underlying structures of mathematics

- Geometric thinking involves developing, attending to, and learning how to work with imagery.
- Decomposing and rearranging provide a geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one.
- Symmetry provides a powerful way of working geometrically.


## Essential Questions

- What types of symmetry can be found around you?
- How can transformations be described algebraically?
- How can you use a line to generate the reflection image of a figure over that line?
- What are the coordinate rules for reflections over the $x$-axis and $y$ axis, dilations centered at the origin, and translations? Why do these rules make sense geometrically?
- Why do these rules make sense geometrically?
- How do you determine the angles of rotation for a figure that has rotational symmetry?
- What does it mean for figures to be similar and/or congruent?


## Common Core State Standards

## GEOMETRY

## Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G. 1

Verify experimentally the properties of rotations, reflections, and translations:
8.G.1a

Lines are taken to lines, and line segments to line segments of the same length.
8.G.1b

Angles are taken to angles of the same measure.

## 8.G.1c

Parallel lines are taken to parallel lines.

## 8.G. 2

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
8.G. 3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

## 8.G. 4

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
8.G. 5

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

## Unit 4 - Linear Relationships, 7 weeks top

Students generalize their knowledge of proportional situations to linear relationships and connect the constant of proportionality seen in tables and graphs to the rate of change (slope). They also represent linear relationships using tables, equations, and graphs. They write equations in $y=m x+b$ form, and learn how the slope $(m)$ portrays the relationship between variables. Scatterplots of collected data are used to determine the relationship between the variables. In the case of a linear-like relationship, a line of best fit is applied. Students will represent linear functions by using spreadsheets and using graphing calculators or computer software. Questions about linear relationships in real-world and mathematical contexts are solved using a variety of methods including tables, graphs, and equations. Linear functions are contrasted with directly proportional patterns of change and inverse relations.

## Big Ideas

The central organizing ideas and underlying structures of mathematics

- Functions are single-valued mappings from one set-the domain of the function- to another- its range.
- A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model.
- Linear functions are characterized by a constant rate of change. Reasoning about the similarity of "slope" triangles allows deducing that linear functions have a constant rate of change and a formula of the type $f(x)=m x+b$.
- Functions can be represented in various ways, including through algebraic means (e.g., equations), graphs, word descriptions, and tables.
- One variable equations are the foundation for Algebra and solving them is rooted in mathematical properties (distributive, addition property of equality, multiplication property of equality).


## Essential Questions

- What is the relationship that describes a function?
- How can functions be used to find the output from a given input?
- How can equations, graphs, word descriptions, and tables describe a function?
- What is a rate and how does is it related to proportional reasoning?
- How can you use mathematical properties to solve one-variable equations?


## Common Core State Standards

## EXPRESSIONS AND EQUATIONS

## Understand the connections between proportional relationships, lines, and linear equations. <br> 8.EE. 5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
8.EE. 6

Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y=m x$ for a line through the origin and the equation $y=m x+b$ for a line intercepting the vertical axis at $b$..

## Analyze and solve linear equations

8.EE. 7

Solve linear equations in one variable.


#### Abstract

8.EE.7a

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x=a$, $a=a$, or $a=b$ results (where $a$ and $b$ are different numbers).

\section*{8.EE.7b}

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.


## FUNCTIONS

## Define, evaluate, and compare functions.

## 8.F. 1

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
8.F. 2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

## 8.F. 3

Interpret the equation $y=m x+b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A=s^{2}$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1),(2,4)$ and ( 3,9 ), which are not on a straight line.

## Use functions to model relationships between quantities.

## 8.F. 4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two ( $x, y$ ) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.F. 5

Describe qualitatively the functional relationship between two quantities by analyzing a graph, (e.g. where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

## Unit 5 - Systems of Linear Relationships, 5 weeks top

The focus of this unit is on developing a conceptual understanding of the connections between algebra and geometry in viewing systems of equations. Students explore situations where multiple solutions are possible and write equations to represent these situations. Graphing technology is used to explore solutions to systems of linear equations.

In order to become proficient in solving systems of equations in a variety of ways, students need to become skilled in converting back and forth between the slope-intercept and standard forms of equations. This will allow students to graph equations (even if not given in slope-intercept form) in order to find the solution(s) to the systems. It also allows students to use the substitution method by first solving for x or y in the equations. The other method explored in this unit is the combining equations method. After practicing and comparing these three methods of solving systems, students should be able to make decisions about methods that will be most feasible for a given system.

| Big Ideas | Essential Questions |
| :--- | :--- |

The central organizing ideas and underlying structures of mathematics

- Solving a system equation can be done with tables, graphs and equations.
- The different methods to solve a system of equations can be more efficient than others, based on the situation and context.


## Essential Questions

- How can situations be modeled as a system of linear equations and how to find solutions using all constraints?
- What does it mean when the graphs of two functions intersect?
- What method is most appropriate solve a system of equations (table, graph, or equation)?


## Common Core State Standards

## EXPRESSIONS AND EQUATIONS

## Analyze and solve linear equations and pairs of simultaneous linear equations.

## 8.EE. 7

Solve linear equations in one variable.

## 8.EE.7a

Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $\mathrm{x}=\mathrm{a}$, a $=\mathrm{a}$, or $\mathrm{a}=\mathrm{b}$ results (where a and b are different numbers).

## 8.EE.7b

Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

## 8.EE. 8

Analyze and solve pairs of simultaneous linear equations.

## 8.EE.8a

Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

## 8.EE.8b

Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3 x+2 y=5$ and $3 x+2 y=6$ have no solution because $3 x+2 y$ cannot simultaneously be 5 and 6 .

## 8.EE.8c

Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

## FUNCTIONS

## Use functions to model relationships between quantities.

## 8.F. 2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
8.F. 4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

## Unit 6 - Volume, 4 weeks top

In this unit, students build off their understanding of area to build an understanding of volume. Students will extend their understandings of volume learned in $7^{\text {th }}$ grade (cubes, and right prisms), to further understanding of cones, cylinders, and spheres. Knowing the formulas for the volumes of cones, cylinders and spheres and applying them to solve real-world and mathematical problems is also a part of this unit. Within the volume formulas, both square roots and cube roots may arise in problem solving situations.


## Unit 7 - Patterns in Data, 5 weeks top

In this unit students will be collecting data sets and displaying them in both two-way tables and scatterplots. When observing the scatterplots students will look for patterns of positive, negative, or no association between the variables. They will not calculate correlation coefficients as this will be addressed in Algebra 1. Students will draw in lines of best fit on graphs that appear to have a linear pattern and use those lines and the equations of those lines to make predictions. Informally students will judge the fit of a line by observing how far points are away from the line. The impact of an outlier on the line of best fit will also be informally discussed. When using a two-way table to display data, students will understand the meaning of the individual cells of the table as well as the rows and columns. They will use the table to look for association between the variables represented. Students use relative frequencies to make inferences from the two way tables and when reasonable generalize to a larger population.

> Big Ideas

The central organizing ideas and underlying structures of mathematics.

- Reading understanding, interpreting, and communicating data are critical in modeling a variety of real-world situations, drawing appropriate inferences, making informed decisions, and justifying those decisions.
- The message conveyed by the data depends on how the data is collected, represented, and summarized.
- The results of a statistical investigation can be used to support or refute an argument.


## Common Core State Standards

## STATISTICS AND PROBABILITY

## Investigate patterns of association in bivariate data.

## 8.SP. 1

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities.

## 8.SP. 2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

## 8.SP. 3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of $1.5 \mathrm{~cm} / \mathrm{hr}$ as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

## 8.SP. 4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew.

