Fairfield Public Schools

Probability & Statistics 40

PROBABILITY & STATISTICS 40

Critical Areas of Focus

Probability and statistics is recommended for those who want an elective that will be beneficial to many academic, medical, social science, and business careers. Statistics topics studied include: describing data with graphs, distributions, histograms and other graphical techniques, and measures of center and spread. Probability topics include: probability rules, probability distributions – discrete and normal. Additional topics include: sampling design, experimental design, sampling distributions, linear regression, and an introduction to inference testing and confidence intervals. Probability & Statistics is an excellent option for students anticipating statistics requirement in college.

1. Unit one focuses on acquiring data, understanding measurement, and critiquing data. Student will learn good sampling techniques to conduct experiments and studies to give results about a given population that is as accurate as possible. In choosing a sampling method it is important to avoid bias and other sources of error by properly using random sampling and through careful design of a sample survey. A properly designed and executed experiment can give good evidence for causation. Control and randomization are the most important aspects of the statistical design of experiments, without them, data collected can be misleading and lead to invalid results.

2. In unit two, the student will be able to describe patterns of univariate and bivariate data as well as observations that do not fit those patterns. Students will learn to organize sets of data into graphs, calculate numerical summaries, and then critically analyze data by describing, interpreting, and comparing important features. Choosing the most appropriate data display and quantitative description of data is an important skill in correctly analyzing and comparing data.

3. In this third unit, the students will apply probability principles to calculate likelihoods, to build probability models, and to calculate expected values. Students will apply the multiplication principle, counting techniques, tree diagrams, simulations, and other probability rules. Differentiation will be made between discrete and continuous random variables and the rules and properties that apply to both.

4. In the fourth unit, students will be introduced to hypothesis testing and confidence intervals and they will use this concept to estimate population parameters and draw conclusions about situations.

Pacing Guide										
1st Marking Period			2nd Marking Period			3rd Marking Period			4th Marking Period	
September	October	November	December	January	February	March	April	May	June	
Unit 1 **Producing Data** 9 weeks			Unit 2 **Organizing Data** 11 weeks			Unit 3 **Probability and Simulation** 9 weeks		Unit 4 **Inference** 7 weeks		

Course Overview		
Central Understandings Insights learned from exploring generalizations through the essential questions. (Students will …) • Observe and describe patterns and departures from patterns. • Plan and conduct a study using samples, experiments, and simulations. • Explore random phenomenon using the normal distribution, probability, and simulation. • Use statistical inference to make conclusions with confidence. • Estimate population parameters and test hypotheses.	**Essential Question** • How can collecting, validating, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?	**Assessments** Assessments will be both formative and summative of various forms including homework, in class work, projects, and tests.

Content Outline	**Standards**
I. Unit 1 – Producing Data II. Unit 2 – Organizing Data III. Unit 3 – Probability and Simulation IV. Unit 4 – Inference	The standards referenced in this curriculum are based on AP and California Statistics standards.

Statistics Standards for Mathematical Practice The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:	
Standards	***Explanations and Examples***
1. Make sense of problems and persevere in solving them.	Students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of mathematical concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. Reason abstractly and quantitatively.	This practice standard refers to one of the hallmarks of mathematical reasoning, the process of de-contextualization and contextualization. Much of statistics involves determining the appropriateness of applying abstract assumptions to a real, specific circumstance. For example, students must validate when they apply the assumption of a normal distribution.
3. Construct viable arguments and critique the reasoning of others.	In Statistics, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	Statistics uses disciplined mathematical principles to create descriptive and predictive models based on available data. For example, students apply regression analysis to make prediction of one variables value from another.
5. Use appropriate tools strategically.	Statistics provides students with a menu of analytic techniques and the principles for choosing the correct technique for any given situation. For example, statistics students learn to choose when to describe a distribution with mean and standard deviation or the five number summary.
6. Attend to precision.	In Statistics the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways. For example, students must address the meaningfulness of the difference between the alpha and p-values.
7. Look for and make use of structure.	In Statistics students should employ common structural patterns in data as the basis of describing the information within univariate and bivariate distributions. For example, students are characterize a univariate distribution by discussing its center, shape, spread, and outliers.
8. Look for and express regularity in repeated reasoning.	Statistics provides employs a variety of distribution to describe common patterns natural phenomena. For example, the normal distribution is frequently assumed to describe a data set, allowing statisticians to employ a menu of regular calculations, tools, and rules.

Unit 1 – Producing Data, 9 weeks top

Unit one focuses on acquiring data, understanding measurement, and critiquing data. Students will learn good sampling techniques to conduct experiments and studies to give results about a given population that is as accurate as possible. In choosing a sampling method it is important to avoid bias and other sources of error by properly using random sampling and through careful design of a sample survey. A properly designed and executed experiment can give good evidence for causation. Control and randomization are the most important aspects of the statistical design of experiments, without them, data collected can be misleading and lead to invalid results.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Observational studies and controlled experiments allow conclusions regarding the population based on data from a carefully selected group of subjects. • Well-designed observational studies can provide information about the characteristics of the population whereas randomized controlled experiments can provide information about treatment effects on the population.	• How do you design and carry out an experiment to allow valid conclusion of causation? • How do you design and carry out a survey to minimize bias and random error? • How do you characterize the results of a study to reflect both the basic findings and appropriate qualifications?

Statistics Standards

PRODUCING DATA

PD-1
Understand different methods of data collection; for example a census, sample survey, experiment and observational study.

PD-2
Critique the reliability, validity, accuracy of measurements.

PD-3
Design, conduct and interpret surveys and experiments.

PD-4
Differentiate between the many sources of bias in sampling and surveys and determine how to avoid them.

PD-5
Summarize survey results with a confidence statement.

PD-6
Determine the sample size corresponding to a given margin of error for a 95% confidence level using the $1/\sqrt{n}$ approximation.

PD-7
Distinguish between different sampling methods including simple random sampling, stratified random sampling, and other methods.

PD-8
Apply the terms of treatments, control groups, and subjects.

PD-9
Understand the principles of good experimental design: control of variables, random assignments, and replication.

PD-10
Apply techniques for addressing bias and confounding, including placebos and blinding in experiments.

PD-10
Differentiate between a basic completely randomized design and other designs.
PD-11
Understand the limitations on generalizing of results and the types of conclusions that can be drawn from observational studies, experiments, and surveys.
PD-12
Critique ethical issues involved in the design of a study.

Unit 2 – Organizing Data, 11 weeks top

In unit two, the student will be able to describe patterns of univariate and bivariate data as well as observations that do not fit those patterns. Students will learn to organize sets of data into graphs, calculate numerical summaries and then critically analyze data by describing, interpreting, and comparing important features. Choosing the most appropriate data display and quantitative description of data is an important skill in correctly analyzing and comparing data. In addition to describing a relationship between variables numerically and graphically, students will address the distinction between association and causation.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Complete analysis of data makes use of correct terminology, varied graphs and appropriate numerical techniques to study patterns and departures from patterns.	• Which calculations provide the most appropriate characterization of distribution? • How do you fully describe a data set in context, using appropriate terminology, calculations, and graphs? • How can technology display and create models? • How do you create a model for bivariate data and how do you describe, interpret and analyze the model? • How can you use the properties of a normal distribution to analyze a data set?

Statistics Standards

ORGANIZING DATA

OD-1
Know the definitions of the mean, median, and mode of distribution of data and how to compute each of them in particular situations using formulas and/or technology.

OD-2
Organize and compare univariate data.

a) Organize data using a number of different methods, including frequency tables, histograms, standard line graphs and bar graphs, stem-and-leaf displays, and box plots.
b) Describe the center, shape, spread and unusual features (outliers) of univariate data.
c) Graphically compare distributions of univariate data.

OD-3
Explore categorical data using bar charts, pie charts, and frequency tables.

OD-4
Determine the mean and the standard deviation of a normally distributed random variable.

OD-5
Understand the meaning of the standard deviation of a distribution of data and compute it using technology.

OD-6
Describe the form, strength, and direction of bivariate data as displayed in a scatterplot.

OD-7
Find the line of best fit to a given distribution of data by using least squares regression.

OD-8
Analyze the relationship characterized by a least squares regression, discussing possible relationships of variables and interpreting intercept, slope, r, and r^2.

OD-9
Understand properties of the normal distribution and how to use, 68-95-99.7 rule, tables, and technology to solve problems.

Unit 3 – Probability, 9 weeks top

In this third unit, the students will apply probability principles to calculate likelihoods, to build probability models, and to calculate expected values for both discrete and continuous variables. Students will apply the multiplication principle, counting techniques, tree diagrams, simulations, and other probability rules. The techniques will be applied to real world decision making.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Probability is the tool used for anticipating future outcomes based on past events, theoretic speculation, or simulation. • Expected value is a key tool for rational decision making. • Simulation allows us to overcome difficulties with other means of calculating probabilities and expected values. • The normal distribution can be applied to many real life situations to solve probability problems.	• How do you use probability rules to evaluate chance behavior in real world contexts? • How can you use expected value for decision making? • How can simulation be used to calculate expected values and probabilities? • How can technology be applied to create and interpret models?

Statistics Standards

PROBABILTIY

P-1
Identify a random process.

P-2
Calculate a probability based on observed proportions, simulation, or theoretical analysis.

P-3
Solve probability problems using the rules for addition (“or”) and multiplication (“and”) rules.

P-4
Understand properties of the normal distribution and how to use tables and technology to solve probability problems.

P-5
Know the role of the “Law of Large Numbers” in improving estimates of probabilities.

P-6
Know the definition of the mean (expected value) of a discrete random variable and determine the mean for a particular set of random events.

P-7
Understand and differentiate probability distributions.

P-8
Design and execute simulations to calculate probabilities and expected values.

Unit 4 – Inference, 7 weeks top

In the fourth unit, students will be introduced to hypothesis testing and confidence intervals and they will use this concept to estimate population parameters and draw conclusions about situations. Significance testing is used to help make a judgment about a claim by addressing the question, "Can the observed difference be attributed to chance?" The students will learn that the value of conclusions from an inference procedure relies on the disciplined application of the assumption and the precision of the calculations involved.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• The foundation of an inference procedure is the hypotheses, null and alternative, which are determined by carefully considering the question of interest. • The p-value determines acceptance of the null hypothesis or rejection of it in favor of the alternative hypothesis based on the likelihood of the observed result. • There are many inference procedures to select from based on the nature of the problem at hand.	• How do you use inferential models to draw statistically significant conclusions from data and make inferences about populations? • How can the language of statistics be used to communicate mathematical ideas coherently and precisely? • How can technology be applied to create and interpret models? • How can improperly applied inference procedures lead to bad conclusions?

Statistics Standards

INFERENCE

I-1

Formulate appropriate null and alternative hypothesis and properly choose a test to conduct in order to reach a conclusion regarding the null and alternative.

a) State hypotheses using proper notation.
b) Determine whether the test is one-sided or two-sided based on the problem setting.

I-2

Select an appropriate testing procedure (i.e. confidence interval or z-test)

I-3

Determine and interpret confidence intervals on proportions and means.

a) Determine confidence intervals for a simple random sample from a normal distribution of data.
b) Determine the sample size required for a desired margin of error.
c) Interpret confidence intervals in context of a given situation.

I-4

Use a z-procedure to determine the *p-value* for a proportion or mean for a simple random sample from a normal distribution.

a) Use the *p-value* to draw a conclusion about the null and alternative hypothesis.
b) Interpret the conclusion in context of a given situation.

I-5

Critique an application of inference procedures.