**Mathematics**

# Fairfield Public Schools

## AP Statistics

## AP STATISTICS

### Critical Areas of Focus

AP Statistics is a rigorous course that offers advanced students an opportunity to do college level work in high school. Students will explore four broad conceptual themes: exploring data, planning a study, probability, and statistical inference. The content of the course requires students to use high level problem solving skills to analyze, describe and make conclusions about sets of data. AP Statistics is an excellent option for all students meeting the prerequisites, regardless of their intended college major. It is expected that students in this course will take the AP exam.

1. In unit one, the student will be able to describe patterns of univariate and bivariate data as well as observations that do not fit those patterns. Students will learn to organize sets of data into graphs, calculate numerical summaries and then critically analyze data by describing, interpreting, and comparing important features. Choosing the most appropriate data display and quantitative description of data is an important skill in correctly analyzing and comparing data.

2. Unit two focuses on using good sampling techniques to conduct experiments and studies to give results about a given population that is as accurate as possible. In choosing a sampling method it is important to avoid bias and other sources of error by properly using random sampling and through careful design of a sample survey. A properly designed and executed experiment can give good evidence for causation. Control and randomization are the most important aspects of the statistical design of experiments, without them, data collected can be misleading and lead to invalid results.

3. In this third unit, the students will use probability as a tool to find the distribution of data for a given model. Students will learn to the multiplication principle, counting techniques, and other probability rules to determine the likelihood of different events. Venn and tree diagrams will be used to organize information and to help solve problems. Differentiation will be made between discrete and continuous random variables and the rules and properties that apply to both. Different probability distributions (including the binomial and normal distributions) will be explored and used to solve problems.

4. In the fourth unit, students will be introduced to hypothesis testing and confidence intervals and they will use this concept to estimate population parameters and draw conclusions about situations. Statistical inference allows us to draw conclusions about one or more parameters of a population. Correctly choosing and carrying out an appropriate test such as Chi-square, one sample t, two proportion z and others in important in enabling us to correctly draw practical conclusions regarding the data.

| Pacing Guide | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1st Marking Period | | | 2nd Marking Period | | | 3rd Marking Period | | | 4th Marking Period | |
| September | October | November | December | January | February | March | April | May | June | |
| Unit 1<br><br>**Exploring Data**<br><br>9 weeks | | | Unit 2<br><br>**Planning a Study**<br><br>4 weeks | | Unit 4<br><br>**Probability**<br><br>9 weeks | | Unit 5<br><br>**Inference**<br><br>12 weeks | | | |

| Course Overview | | |
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| **Central Understandings**<br>Insights learned from exploring generalizations through the essential questions. (Students will …)<br>• Observe and describe patterns and departures from patterns.<br>• Plan and conduct a study using samples, experiments, and simulations.<br>• Explore random phenomenon using probability and simulation.<br>• Use statistical inference to make conclusions with confidence.<br>• Estimate population parameters and test hypotheses. | **Essential Question**<br>• How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions? | **Assessments**<br>• Formative Assessments<br>• Summative Assessments |

| **Content Outline** | **Standards** |
| --- | --- |
| I. Unit 1 – Exploring Data<br>II. Unit 2 – Planning a Study<br>III. Unit 3 – Probability<br>IV. Unit 4 – Inference | The standards referenced in this curriculum are from the AP and California Statistics standards. |

| **Statistics Standards for Mathematical Practice**<br>The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to: | |
|---|---|
| ***Standards*** | ***Explanations and Examples*** |
| **1. Make sense of problems and persevere in solving them**. | Students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" |
| **2. Reason abstractly and quantitatively.** | This practice standard refers to one of the hallmarks of mathematical reasoning, the process of de-contextualization and contextualization. Much of statistics involves determining the appropriateness of applying abstract assumptions to a real, specific circumstance. For example, students must validate when they apply the assumption of a normal distribution. |
| **3. Construct viable arguments and critique the reasoning of others.** | In Statistics, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking. |
| **4. Model with mathematics.** | Indeed, other mathematical practices in Statistics might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together. |
| **5. Use appropriate tools strategically.** | Students consider available tools such as spreadsheets, a function modeling language, graphing tools and many other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics. |
| **6. Attend to precision.** | In Statistics, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways. |
| **7. Look for and make use of structure.** | In Statistics students should employ common structural patterns in data as the basis of describing the information within univariate and bivariate distributions. For example, students are characterize a univariate distribution by discussing its center, shape, spread, and outliers. |
| **8. Look for and express regularity in repeated reasoning**. | Statistics provides employs a variety of distribution to describe common patterns natural phenomena. For example, the normal distribution is frequently assumed to describe a data set, allowing statisticians to employ a menu of regular calculations, tools, and rules. |

## Unit 1 – Exploring Data, 9 weeks top

The student will be able to describe patterns of data as well as observations that do not fit those patterns. During this unit an emphasis will be placed on organizing sets of data into graphs, calculating numerical summaries and then critically analyzing data by describing, interpreting, and comparing important features. In examining distributions of data, students should be able to detect important characteristics, such as shape, location, variability and unusual values. From careful observations of patterns in data, students can generate conjectures about relationships among variables. The notion of how one variable may be associated with another permeates almost all of statistics, from simple comparisons of proportions through linear regression. The difference between association and causation must accompany this conceptual development throughout.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • Exploratory analysis of data makes use of graphical and numerical techniques to study patterns and departures from patterns. Emphasis should be placed on interpreting information from graphical and numerical displays and summaries. | • How are appropriate techniques, tools, and formulas used in Statistics to draw conclusions?<br>• How can the language of Statistics be used to communicate mathematical ideas coherently and precisely?<br>• How can technology be applied to create and interpret models?<br>• How do you describe a distribution of data numerically and graphically?<br>• How do you create a model for bivariate data and how do you describe, interpret and analyze the model? |

### Statistics Standards

**EXPLORATION OF DATA**

**ED-1** Calculate summary measures of data

a) Know the definitions of the mean, median, and mode of distribution of data and how to compute each of them in particular situations using formulas and/or technology.
b) Understand the effect of changing units and applying transformations on data on the summary measures.

**ED-2**

Organize and compare univariate data.

a) Organize data using a number of different methods, including frequency tables, histograms, standard line graphs and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots.
b) Describe the center, shape, spread and unusual features (outliers) of univariate data.
c) Compare distributions of univariate data. For example, using back-to-back stem plots

**ED-3**

Explore and compare categorical data using bar charts, frequency tables, and marginal and joint frequencies of two-way tables.

**ED-4**

Determine the mean and the standard deviation of a normally distributed random variable.

**ED-5**

Compute the variance and the standard deviation of a distribution of data using formulas and/or technology. Understand the meaning of variance and standard deviation of a data set.

**ED-6** Organizing and comparing bivariate data.

a) Analyze patterns in bivariate data using scatterplots
b) Find the line of best fit to a given distribution of data by using least squares regression.
c) Analyze residual plots for outliers and influential points
d) Use transformations to achieve linearity. For example, apply logarithmic and power transformations.

## Unit 2 – Planning a Study, 4 weeks top

This unit focuses on using good sampling techniques to conduct experiments and studies to give results about a given population that is as accurate as possible. If data are to be collected to provide an answer to a question of interest, a careful plan must be developed. Both the type of analysis that is appropriate and the nature of conclusions that can be drawn from that analysis depend in a critical way on how the data was collected. Collecting data in a reasonable way, through either sampling or experimentation, is an essential step in the data analysis process.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • Statistical models extend mathematical models by describing variability around the structure.<br>• Observational studies, including surveys, provide information about the characteristics of a population or sample, whereas controlled experiments and simulations provide information about treatment effects.<br>• Random selection tends to produce samples that are representative of the population, permitting generalization from the sample to the larger population and also allowing the uncertainty in estimates to be quantified. | • How do you design and carry out an experiment to produce valid information?<br>• How do you use chance in random sampling and randomized comparative experiments to simulate random behavior?<br>• How can technology be applied to carry out simulations?<br>• How can technology be used in randomizing experiments and surveys? |

### Statistics Standards

**PLANNING A STUDY**

**PAS-1**
Understand different methods of data collection; for example a census, sample survey, experiment and observational study.

**PAS-2**
Design, conduct and interpret surveys and experiments.

**PAS-3**
Differentiate between the many sources of bias in sampling and surveys and determine how to avoid them.

**PAS-4**
Distinguish between different sampling methods including simple random sampling, stratified random sampling, and cluster sampling.

**PAS-5**
Know the definitions of treatments, control groups, experimental units, random assignments, and replication.

**PAS-6**
Understand various sources of bias and confounding, including placebo effect and blinding in experiments.

**PAS-7**
Differentiate between different experiment designs including completely randomized design, randomized block design and matched pairs design.

**PAS-8**
Create and conduct a simulation and understand the role that simulations play in collecting data.

**PAS-9**
Understand the limitations on generalizing of results and the types of conclusions that can be drawn from observational studies, experiments, and surveys.

## Unit 3 – Probability, 9 weeks top

Exploring random phenomena using probability and simulation. In this unit, the students will use probability as a tool to find the distribution of data for a given model. Random phenomena are not haphazard: they display an order that emerges only in the long run and is described by a distribution. The mathematical description of variation is central to statistics. The probability required for statistical inference is not primarily axiomatic or combinatorial but is oriented toward using probability distributions to describe data.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • Probability is the tool used for anticipating what the distribution of data should look like under a given model.<br>• The Law of Large Numbers allows us to make generalizations about data in the 'long run.'<br>• The Central Limit Theorem tells us that the sample mean has an approximately normal distribution when the sample is large.<br>• The Normal Distribution can be applied to many real life situations to solve probability problems. | • How do you use probability rules to evaluate chance behavior in real world contexts?<br>• How do you anticipate what a distribution of data should look like under a given model?<br>• How do you describe a probability distribution numerically and graphically?<br>• How can technology be applied to create and interpret models? |

### Statistics Standards

**PROBABILITY**

**P-1**
Solve probability problems with finite sample spaces by using the rules for addition, multiplication, and complementation for probability distributions and understand the simplifications that arise with independent events.

**P-2**
Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.

**P-3**
Demonstrate an understanding of the notion of discrete random variables by using this concept to solve for the probabilities of outcomes, such as the probability of the occurrence of five or fewer heads in 14 coin tosses.

**P-4**
Understand the notion of a continuous random variable and interpret the probability of an outcome as the area of a region under the graph of the probability density function associated with the random variable.

**P-5**
Know the "Law of Large Numbers" and how it relates to probability problems.

**P-6**

Means and Standard Deviations of discrete random variables

a) Know the definition of the mean (expected value) of a discrete random variable and determine the mean for a particular discrete random variable.
b) Know the definition of the variance of a discrete random variable and determine the variance for a particular discrete random variable.
c) Understand and be able to find the result of linear transformations on mans and standard deviations of random variables and combining independent random variables.

**P-7**

Know the Central Limit Theorem and use it to obtain approximations for probabilities in problems of finite sample spaces in which the probabilities are distributed binomially.

**P-8**

Probability Distributions

a) Understand properties of the normal distribution and how to use tables and technology to solve probability problems.
b) Differentiate between the normal distribution and the t-distribution and be able to appropriately use each.
c) Differentiate between sampling distributions of a sample proportion, a sample mean, the difference of two independent sample proportions, the difference of two independent sample means and of matched pair means and be able to appropriately use each.

## Unit 4 – Inference, 12 weeks top

In this unit, students will be introduced to hypothesis testing and confidence intervals and they will use this concept to estimate population parameters and draw conclusions about situations. Models and data interact in statistical work: models are used to draw conclusions from data, while the data are allowed to criticize and even falsify the model through inferential and diagnostic methods. Inference from data can be thought of as the process of selecting a reasonable model, including a statement in probability language, of how confident one can be about the selection.

| **Big Ideas**<br>The central organizing ideas and underlying structures of mathematics | **Essential Questions** |
|---|---|
| • Statistical inference guides the selection of appropriate models.<br>• A hypothesis test involves choosing between two competing hypothesis – the null and the alternative hypotheses.<br>• The alternative hypothesis is determined by the statistical question of interest.<br>• The null hypothesis is rejected in favor of the alternative hypothesis if the sample data provide convincing evidence against the null hypothesis.<br>• The *p-value* measures surprise.<br>• Hypothesis tests do not always lead to a correct decision. | • How do you use inferential models to analyze experimental designs, draw statistically significant conclusions form data, and make inferences about populations?<br>• How can the language of Statistics be used to communicate mathematical ideas coherently and precisely?<br>• How can technology be applied to create and interpret models? |

### Statistics Standards

**INFERENCE: CONCLUSIONS WITH CONFIDENCE**

**I-1**
Understand the statistic of a distribution of values, the sampling distribution of a statistic, and of the variability of a statistic.

**I-2**
Know basic facts concerning the relation between the mean and the standard deviation of a sampling distribution and the mean and the standard deviation of the population distribution.

**I-3** Confidence Intervals.

a) Determine confidence intervals for a simple random sample from a normal distribution of data.
b) Determine the sample size required for a desired margin of error.
c) Interpret confidence intervals in context of a given situation.

**I-4**
Formulate appropriate null and alternative hypothesis and properly choose a test to conduct in order to reach a conclusion regarding the null and alternative.

**I-5**
Determine the *p-value* for a statistic for a simple random sample from a normal distribution. Use the *p-value* to draw a conclusion about the null and alternative hypothesis and interpret the conclusion in context of a given situation.

**I-6**
Understand and determine different types of error (Type I and Type II) and calculate the power of a test.

**I-7**
Understand what a chi-square distribution is and how to use the chi- square tests for goodness of fit, homogeneity and independence. Be able to use one- and two-way tables.

**I-8**
Conduct a test for the slope of a least-squares regression line and interpret the results.