Fairfield Public Schools

PRE-CALCULUS 41

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Critical Areas of Focus

Pre-calculus combines the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students intending to study calculus, physics, and other sciences, and/or engineering in college. Because the standards for this course are (+) standards, students selecting this Pre-Calculus course should have met the college and career ready standards. The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. For this high school Pre-Calculus course, instructional time should focus on four critical areas: (1) extend work with complex numbers; (2) expand understanding of logarithmic, exponential, and trigonometric functions; (3) use characteristics of polynomial and rational functions to sketch graphs of those functions; and (4) perform operations with vectors and matrices.

1) Students continue their work with complex numbers. They perform arithmetic operations with complex numbers and represent them and the operations on the complex plane. Students investigate and identify the characteristics of the graphs of polar equations, using graphing tools. This includes classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations. Students translate the geometric figures.

2) Students expand their understanding of functions to include logarithmic, exponential, and trigonometric functions. They investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This includes the role of e, natural and common logarithms, laws of exponents and logarithms, and the solutions of logarithmic and exponential equations. Students model periodic phenomena with trigonometric functions and prove trigonometric identities. Other trigonometric topics include reviewing unit circle trigonometry, proving trigonometric identities, solving trigonometric equations, and graphing trigonometric functions.

3) Students investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. They determine zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. They deepen their understanding of the Fundamental Theorem of Algebra.

4) Students perform operations with vectors and matrices and solve practical problems using vectors and matrices. This includes the following topics for vectors: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components and for matrices: operations of addition, subtraction, scalar multiplication, dimensional multiplication, and inversion.

Pacing Guide									
1st Marking Period		2nd Marking Period			3rd Marking Period		4th Marking Period		
September	October	November	December	January	February	March	April	May	June

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Rational and Exponential Functions	**Trigonometry**	**Vectors and Matrices**	**Complex Numbers**	**Introduction to Limits**
8 weeks	7 weeks	8 weeks	4 weeks	4 weeks

Course Overview

Central Understandings	Essential Questions	Assessments
Insights learned from exploring generalizations through the essential questions. (Students will understand that…) • Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies. • Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies. • Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools, and technologies. • Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.	• How do patterns and functions help us describe data and physical phenomena and solve a variety of problems? • How are quantitative relationships represented by numbers? • How do geometric relationships and measurements help us to solve problems and make sense of our world? • How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?	• Formative Assessments • Summative Assessments

Content Outline	**Standards**
I. Unit 1 – Rational and Exponential Functions II. Unit 2 – Trigonometry III. Unit 3 – Vectors and Matrices IV. Unit 4 – Complex Numbers V. Unit 5 – Introduction to Limits	Connecticut Common Core State Standards are met in the following areas: • ***Number and Quantity*** • ***Algebra*** • ***Geometry*** • ***Functions***

Pre-Calculus Standards for Mathematical Practice

The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

Standards	***Explanations and Examples***
1. Make sense of problems and persevere in solving them.	Students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
2. Reason abstractly and quantitatively.	This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations to reveal properties of the problems.
3. Construct viable arguments and critique the reasoning of others.	In Pre-Calculus, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, graphs and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	Indeed, other mathematical practices in Pre-Calculus might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.
5. Use appropriate tools strategically.	Students consider available tools graphing calculators and other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics.
6. Attend to precision.	In Pre-Calculus, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways.
7. Look for and make use of structure.	In Pre-Calculus, converting rectangular equations describing the path of an object can be restructured into parametric form to allow the description of the objects position relative to horizontal position, vertical position, and time. A student can then tell not only the positions the object will occupy but also when it is in each position.
8. Look for and express regularity in repeated reasoning.	Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism. For example, students should be able to see a pattern in the development of the points on the unit circle through their reasoning of 30-60-90 and 45-45-90 triangles.

Unit 1 – Rational and Exponential Functions, 8 weeks top

In this unit the students will learn to analyze and to identify all the aspects of rational and exponential functions, using complex numbers and the fundamental theorem of algebra. Students will be able to graph and transform functions, using known aspects of functions and a table of values. Students will use exponential functions for modeling growth and decay, as with populations and radioactivity. Rational functions will be used to explore the social, behavioral, and natural sciences.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Analysis of rational functions requires deconstruction of the underlying polynomial functions. • The fundamental theorem of algebra states that every non-constant single-variable polynomial with complex coefficients has at least one complex root.	• Why and how are complex numbers used? • What are the graphical features of the relationship? • What do those features mean in a modeling problem? • When and how can composition be used in constructing and analyzing functions? • How can a rational expression be re-expressed as a sum or difference of fractions?

Common Core State Standards

NUMBER AND QUANTITY

The Complex Number System

Use complex numbers in polynomial identities and equations.

N-CN.8 (+)

Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

N-CN.9 (+)

Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

ALGEBRA

Arithmetic with Polynomials and Rational Expressions

Use polynomial identities to solve problems

A-APR.5 (+)

Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rewrite rational expressions

A-APR.7 (+)

Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

FUNCTIONS

Interpreting Functions

Analyze functions using different representations

F-IF.7

Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F-BF.Fairfield.7 (+)

Use secant line to analyze rate of change approaching turning points and asymptotes.

F-IF.9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions

Build a function that models a relationship between two quantities

F-BF.1

Write a function that describes a relationship between two quantities.

c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

Build new functions from existing functions

F-BF.4

Find inverse functions.

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

F-BF.5 (+)

Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Unit 2 – Trigonometry, 7 weeks [top]

In this unit the students learn that the trigonometric functions are far more than tools for finding the dimensions of right triangles. The students come to understand the functions' value in analyzing the relationships of circles and right triangles and their value in portraying natural phenomena. Also, this more advanced work with trigonometric functions improves foundations in several areas critical in calculus, establishing identities, and working with functions with restricted domains. As trigonometric functions will arise frequently in more advanced work for these and other reasons, beyond its specific lessons this unit also has value in providing greater depth and breadth of familiarity with these important functions.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• For angles between 0° and 90°, the trigonometric functions can be defined as ratios of side lengths in right triangles. These functions are well defined because the ratios of side lengths are equivalent in similar triangles. • For general angles, the sine and cosine functions can be viewed as the *y*- and *x*-coordinates of points on circles or as the projection of circular motion onto the *y*- and *x*-axes. • The unit circle captures many fundamental relationships related to right triangles by modeling periodicity of trigonometric functions and allows mapping from degree to radian measure • Parametric equations allow analysis of 2 dimensional position and time.	• How do the features of a trigonometric function correspond to various natural phenomena? • What are the fundamental relationships between the trigonometric functions? • How can the trigonometric functions be represented with right triangles in circles? • How can I depict the position of an object at any given time?

Common Core State Standards

FUNCTIONS

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle

F-TF.3 (+)
Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

F-TF.4 (+)
Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

F-TF.6 (+)
Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.7 (+)
Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

F-TF.9(+)

Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Parametric equations

F-BF.Fairfield.8 (+)

Describe a path and key features of the path by giving the x and y coordinates of the path as functions of a third parameter such as time.

GEOMETRY

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles

G-SRT.9 (+)

Derive the formula $A = \frac{1}{2} absin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.10 (+)

Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11 (+)

Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Unit 3 – Vectors and Matrices, 8 weeks [top]

In this unit, students will learn to represent magnitude (or size) and direction using vectors, and represent data and linear relationships using matrices. Students use basic operations on and with vectors and matrices to solve problems.

Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Vector analysis can represent various physical phenomena. • Matrices allow us to represent multidimensional problems.	• What does a vector represent? • How are operations with vectors and matrices different than operations on scalars? • How can vector operations solve linear systems?

Common Core State Standards

NUMBER AND QUANITY

Vector Quantities and Matrices

Represent and model with vector quantities.

N-VM.1 (+)

Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, *v*).

N-VM.2 (+)

Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N-VM.3 (+)

Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

N-VM.4 (+)

Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction **v** – **w** as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.5 (+)

Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).

b. Compute the magnitude of a scalar multiple c**v** using ||c**v**|| = |c|v. Compute the direction of c**v** knowing that when |c|v ≠ 0, the direction of c**v** is either along **v** for (c > 0) or against **v** (for c < 0).

F-BF.Fairfield.6 (+)
Decompose a rational function into partial fractions including rational functions with irreducible quadratic roots in the denominator.

Perform operations on matrices and use matrices in applications.

N-VM.6 (+)
Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM.7 (+)
Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

N-VM.8 (+)
Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9 (+)
Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM.10 (+)
Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N-VM.11 (+)
Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

ALGEBRA

Solve systems of equations

A-REI.8 (+)
Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9 (+)
Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Unit 4 – Complex Numbers 4 weeks top	
In this unit students learn to represent and to analyze complex numbers in the complex plane using both rectangular and polar forms. Further the students will learn to combine this knowledge with their knowledge of matrices to perform rigid transformation.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Complex numbers can be represented in the complex plane. • The complex number *i* is a necessary tool for advanced calculations and geometric manipulations. • The complex numbers are numbers that can be written in the form $a + bi$, where i denotes a square root of -1. • The points in complex plane can be described in polar or rectangular form. • Rigid transformations can be executed using trigonometric functions within matrices.	• What does *i* represent? • What does a complex number represent in a Cartesian plane? • How do we execute a rigid transformation with matrices?

Common Core State Standards

NUMBER AND QUANTITY

Vector Quantities and Matrices

Perform arithmetic operations with complex numbers.

N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

The Complex Number System

Represent complex numbers and their operations on the complex plane.

N-CN.4 (+)
Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5 (+)
Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}i)^3 = 8$ *because* $(-1 + \sqrt{3}i)$ *has modulus 2 and argument 120°.*

N-CN.6 (+)
Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

GEOMETRY(This would be its own unit if we cannot find a book that uses matrices based on trigonometric parameters for transformations)
Congruence
Experiment with transformations in the plane

N-VM.12 (+)
Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

G-CO.2
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.4
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.5
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Unit 5 – Introduction to Limits, 4 weeks [top]	
The concept of limits is fundamental to calculus. This unit establishes the core concept of limits, the basic tools for evaluating them, and simple graphical analysis of limits.	
Big Ideas The central organizing ideas and underlying structures of mathematics	**Essential Questions**
• Limits support the concepts of convergence and continuity.	• What are the upper and lower bounds of a given function, if any? • How do you determine the limit of a function for a value restricted by the domain?
Standards	
NUMBER AND QUANTITY ***Limits*** **Use limits to determine values of functions.** **L.1 (+)** Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. **L.2 (+)** Know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity. (not in PC40) **L.3 (+)** Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions. **L.4 (+)** Use graphing calculators to verify and estimate limits. **L.5 (+)** Prove and use special limits, such as the limits of $\frac{sin(x)}{x}$ and $\frac{1-cos(x)}{x}$ as *x* tends to 0.	