

# Mathematics

# Fairfield Public Schools

## Algebra 12



## ALGEBRA 12

### Critical Areas of Focus

The fundamental purpose of this Algebra 12 course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The critical areas deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout the course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

1. In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this first critical area, students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They compare and contrast linear and non-linear functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
2. By the end of Pre-Algebra, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques. Students also explore systems of equations and inequalities, and they find and interpret their solutions.
3. The third critical area builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.
4. The last critical area, students will consider quadratic functions, comparing the key characteristics of quadratic functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic functions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Pacing Guide									
1st Marking Period		2nd Marking Period			3rd Marking Period			4th Marking Period	
September	October	November	December	January	February	March	April	May	June
Unit 2 <b>Modeling with Functions</b>  5 weeks	Unit 3 <b>Linear Functions</b>  4 weeks	Unit 4 <b>Solving One Variable Equations and Inequalities</b>  4 weeks	Unit 5 <b>Solving Two Variable Equations and Inequalities</b>  3 weeks	MIDTERM	Unit 6 <b>Linear Modeling</b>  3 weeks	Unit 7 <b>Quadratic Functions</b>  5 weeks	Unit 8 <b>Quadratic Equations</b>  6 weeks	Unit 9 <b>Statistics</b>  2 weeks	FINAL

Course Overview		
<p><b>Central Understandings</b></p> <p>Insights learned from exploring generalizations through the essential questions. (Students will understand that...)</p> <ul style="list-style-type: none"> <li>Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies.</li> <li>Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.</li> <li>Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools, and technologies.</li> <li>Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.</li> </ul>	<p><b>Essential Questions</b></p> <ul style="list-style-type: none"> <li>How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?</li> <li>How are quantitative relationships represented by numbers?</li> <li>How do geometric relationships and measurements help us to solve problems and make sense of our world?</li> <li>How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?</li> </ul>	<p><b>Assessments</b></p> <ul style="list-style-type: none"> <li>Formative Assessments</li> <li>Summative Assessments</li> </ul>

**Content Outline**

- I. [Unit 1](#) – Modeling with Functions
- II. [Unit 2](#) – Linear Functions
- III. [Unit 3](#) – Solving One Variable Equations and Inequalities
- IV. [Unit 4](#) – Solving Two Variable Equations and Inequalities
- V. [Unit 5](#) – Linear Modeling and Project
- VI. [Unit 6](#) - Quadratic Functions
- VII. [Unit 7](#) – Quadratic Equations
- VIII. [Unit 8](#) – Statistics

**Standards**

Connecticut Common Core State Standards are met in the following areas:

- *The Number Systems*
- *Expressions and Equations*
- *Number and Quantity*
- *Algebra*
- *Functions*
- *Statistics and Probability*

### Algebra 12 Standards for Mathematical Practice

The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

<i>Standards</i>	<i>Explanations and Examples</i>
<b>1. Make sense of problems and persevere in solving them.</b>	In Algebra, students solve problems involving equations and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?”
<b>2. Reason abstractly and quantitatively.</b>	This practice standard refers to one of the hallmarks of algebraic reasoning, the process of de-contextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems.
<b>3. Construct viable arguments and critique the reasoning of others.</b>	In Algebra, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
<b>4. Model with mathematics.</b>	Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.
<b>5. Use appropriate tools strategically.</b>	Students consider available tools such as spreadsheets, a function modeling language, graphing tools and many other technologies so they can strategically gain understanding of the ideas expressed by individual content standards and to model with mathematics.
<b>6. Attend to precision.</b>	In Algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely helps students understand the idea in new ways.
<b>7. Look for and make use of structure.</b>	In Algebra, students should look for various structural patterns that can help them understand a problem. For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$ is a practice many teachers refer to as “chunking,” highlights the structural similarity between this expression and $z^2 + 5z + 6$ , leading to a factorization of the original: $((7x) + 3)((7x) + 2)$ .
<b>8. Look for and express regularity in repeated reasoning.</b>	Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism. For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing general formulas that express the cost of each plan for <i>any</i> number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent and make a complete analysis of the two plans.

## Unit 1 – Modeling with Functions, 5 weeks

This initial unit starts with a treatment of quantities as preparation for work with modeling. The work then shifts to a general look at functions with an emphasis on representation in graphs, and interpretation of graphs in terms of a context. More emphasis is placed on qualitative analyses than calculation and symbolic manipulation. Linear and non-linear examples are explored.

### Quantities

A short treatment of the general notion of a "quantity" thought of as a number with a specific unit of measure. Includes unit analysis (dimensional analysis). Examples of simple quantities with standard units of measure; the fundamental dimensions of quantities (length, time, weight, temperature, counts); division of quantities: quotient units; examples of quantities with quotient units: speed, flow rate, frequency, price, density, pressure; quotient units and "rates"; quotient units and unit conversion; unit analysis/dimensional analysis; multiplication of quantities: product units; area and volume as examples of quantities with product units; person-days and kilowatt hours as other examples of product units;

### Functions

A general treatment of the function concept with minimal use of symbolic expressions, and instead emphasis on the idea of a function as a mapping represented in graphs or tables. The functions used in this unit, will be mostly linear and 'baby exponential'. In grade 11, student will thoroughly study exponential functions. But they will be introduced to them here so they can compare two different types of functions. Quadratics or piecewise functions can be used to illustrate the properties of functions. Domain and range; functions defined by graphs and their interpretation; functions defined by tables and their interpretation; properties of particular functions (rate of change, zeros) and their meaning in an application; sums and differences of two functions; product of a function and a constant; vertical shifts and horizontal shifts; equality of two functions vs. values where two functions are equal; equations defined in terms of functions and their solution; functions defined by geometric conditions (projections); functions defined recursively; sequences.

<b>Big Ideas</b>	<b>Essential Questions</b>
<p>The central organizing ideas and underlying structures of mathematics</p> <ul style="list-style-type: none"> <li>• Functions can be represented in multiple ways, including algebraic (symbolic), graphical, verbal, and tabular representations. Links between these different representations are important to study relationships and change.</li> <li>• Functions are a single-valued mapping from one set-the domain of the function-to another-its range.</li> </ul>	<ul style="list-style-type: none"> <li>• How can tables, graphs, and rules relating variables be used to answer real-world and mathematical questions about the relationships between variables?</li> <li>• How does the shape of a graph, the patterns in a table, the parts of the rule or verbal description give clues about the way the variables are related to one another?</li> <li>• What are the advantages and disadvantages of using graphs, tables, descriptions, and algebraic rules to demonstrate the relationship between two variables?</li> <li>• What is a function? How do you recognize a function in various representations?</li> <li>• Given two variables, how do you decide which is the independent variable and which is the dependent variable?</li> </ul>

## Common Core State Standards

### **NUMBER AND QUANTITY**

#### **Quantities**

##### **Reason quantitatively and use units to solve problems.**

###### **N-Q 1**

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

###### **N-Q 2**

Define appropriate quantities for the purpose of descriptive modeling.

###### **N-Q 3**

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

### **FUNCTIONS**

#### **Interpreting Functions**

##### **Understand the concept of a function and use function notation**

###### **F-IF 1**

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y=f(x)$ .

###### **F-IF 2**

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

###### **F-IF 3**

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$*

##### **Interpret functions that arise in applications in terms of the context**

###### **F-IF 4**

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\**

###### **F-IF 5**

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.\**

##### **Analyze functions using different representations**

###### **F-IF 9**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

*Linear, Quadratic, and Exponential Models*

**Construct and compare linear, quadratic, and exponential models and solve problems**

**F-LE 1**

Distinguish between situations that can be modeled with linear functions and with exponential functions.

**F-LE 1a**

Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

**F-LE 1b**

Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

**F-LE 3**

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.



## Unit 2 – Linear Functions, 4 weeks

In general, this unit is a thorough treatment of linear functions of one variable  $f(x) = mx + b$ , which builds off the students' understanding of linear equations from Pre-Algebra. Representation of linear functions using expressions, graphs, and tables; identifying and interpreting the three parameters  $x$ -intercept  $x_0$ ,  $y$ -intercept  $y_0$ , and rate of change  $k$ ; creating expressions for linear functions using each pair of parameters ( $y = mx + b$ ,  $y = k(x - x_0) + y_0$ , and  $Ax + By = C$ ); understanding geometrically why the graph is a line; seeing uniform change as the unique feature of linear functions; modeling a variety of situations using functions; working at in-depth solutions of selected problems; looking at the special properties of pure linear functions  $y = ax$  and their role in representing proportional relationships; linear sequences ("arithmetic" sequences).

### Big Ideas

The central organizing ideas and underlying structures of mathematics

- Linear functions are characterized by a constant average rate of change (or constant additive change).
- Reasoning about the similarity of "slope triangles" allows deducing that linear functions have a constant rate of change and a formula of the type  $f(x) = mx + b$ .

### Essential Questions

- What is the meaning of the slope and  $y$ -intercept of that line in terms of the problem situation?
- What are some methods to solve an absolute value equation?
- How do the table and graph of a linear function relate to the table and graph of the absolute value of the same linear function?

## Common Core State Standards

### **FUNCTIONS**

#### ***Interpreting Functions***

**Interpret functions that arise in applications in terms of the context.**

##### **F-IF 6**

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Just introduce concept — will be covered again in quadratics and in other functions units.)*

##### **F-IF 7**

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

##### **F-IF 9**

Graph linear functions and show intercepts, maxima, and minima.

#### ***Building Functions***

**Build a function that models a relationship between two quantities.**

##### **F-BF 1**

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Only linear functions.)*

##### **F-BF 1a**

Determine an explicit expression, a recursive process, or steps for calculation from a context.

**F-BF 2**

Write arithmetic ... sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

***Linear, Quadratic, and Exponential Models***

**Construct and compare linear, quadratic, and exponential models and solve problems.**

**F-LE 1**

Distinguish between situations that can be modeled with linear functions [and with exponential functions].

**F-LE 1a.**

Prove that linear functions grow by equal differences over equal intervals... over equal intervals.

**F-LE 1b.**

Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.... **(Only linear functions.)**

### Unit 3 – Solving One Variable Equations and Inequalities, 4 weeks

This unit will have students complete a thorough treatment to solution of linear equations in one unknown  $ax + b = cx + d$ . Students will complete two sub-units dealing with (a) solving linear equation and inequalities, and (b) creating linear equations.

#### Solving Linear Equations and Inequalities:

Connecting linear equations to linear functions; solving linear equations both through manipulation of expressions and graphically; understanding the conditions under which a linear equation has no solution, one solution, or an infinite number of solutions; seeing the solution of a linear equation  $ax + b = cx + d$  as involving the intersection of the graphs of two linear functions; also seeing the solution  $x$  as the number where two linear functions have the same value; solving a linear equation step by step and graphing each step; reducing any linear equation to the normal form  $Ax + B = 0$ , and finding the solution  $x = -(B/A)$ ; solving linear inequalities  $ax + b < c$  and  $|ax + b| < c$ , and representing the solution on a number line;

#### Creating Linear Equations:

Solving a wide variety of problems in applied settings; seeing that in order to obtain an equation we have to express the same quantity in two different ways; solving a linear equation in an applied setting to get a number, then replacing one of the numerical parameters in the setting with a parameter, and solving again, this time getting a function of the parameter.

Big Ideas	Essential Questions
The central organizing ideas and underlying structures of mathematics	
<ul style="list-style-type: none"> <li>To obtain a solution to an equation, no matter how complex, always involves the process of undoing the operations with the properties of real numbers (e.g., property of equality, identity, distributive, etc.)</li> </ul>	<ul style="list-style-type: none"> <li>What is an equation?</li> <li>What does equality mean?</li> <li>How can we use linear equations and linear inequalities to solve real world problems?</li> <li>What is a solution set for a linear equation or linear inequality?</li> <li>How can models and technology aid in the solving of linear equations and linear inequalities?</li> <li>How are linear functions and equations related?</li> </ul>

### Common Core State Standards

#### ALGEBRA

Sub-unit 1: Solving Linear Equations

#### *Reasoning with Equations and Inequalities*

**Understand solving equations as a process of reasoning and explain the reasoning.**

##### A-REI 1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**Solve equations and inequalities in one variable.**

##### A-REI 3

Solve linear equations and inequalities and inequalities in one variable, including equations with coefficients represented by letters.

**Represent and solve equations and inequalities graphically.**

**A-REI 11**

Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

Sub-unit 2: Creating Linear Equations

***Creating Equations***

**Create equations that describe numbers or relationships.**

**A-CED 1**

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear functions ....*

**A-CED 3**

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**A-CED 4**

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .

## Unit 4 – Solving Two Variable Equations and Inequalities, 3 weeks

In this unit, students will experience a thorough treatment of linear equations in two unknowns  $ax + by + c = 0$ , including simultaneous solution of two such equations. Seeing the differences among the solution to a single equation in one unknown, solutions to a single equation in two unknowns, and "simultaneous" solutions to two equations in two unknowns; explore briefly linear functions in two variables  $F(x,y) = ax + by + c$  and the three dimensional graph (a plane)  $z = ax + by + c$  of such functions; see the solution to an equation  $ax + by + c = 0$  in two unknowns as the intersection of the graph of this function with the x-y plane; make the analogy with functions and equations of one variable/unknown; explore the symmetric form  $(x/x_0) + (y/y_0) = 1$  of a linear equation in two unknowns and compare equations of conic sections; see the solution of any equation in two unknowns as a line or curve in the x-y plane; find simultaneous solutions to pairs of equations  $ax + by + c = 0$  and  $a'x + b'y + c' = 0$  symbolically; solve many problems set in an applied context using such equations.

Big Ideas	Essential Questions
The central organizing ideas and underlying structures of mathematics	
<ul style="list-style-type: none"> <li>A system of linear equations is an algebraic way to compare two equations that model a situation and find the breakeven point or choose the most efficient or economical plan.</li> <li>Systems of linear equations have different methods to find the solution; each method can be used at any time, but the different methods can be more efficient than others.</li> </ul>	<ul style="list-style-type: none"> <li>What does the number of solutions (none, one or infinite) of a system of linear equations represent?</li> <li>What are the advantages and disadvantages of solving a system of linear equations graphically versus algebraically?</li> <li>Given equations of two lines, how can you determine whether the lines are parallel, perpendicular, intersecting or concurrent?</li> <li>What are the possible types of solutions for a system of two linear equations? Give sample equations that demonstrate these types?</li> </ul>

### Common Core State Standards

#### **ALGEBRA**

##### ***Creating Equations***

**Create equations that describe numbers or relationships.**

##### **A-CED 2**

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

##### **A-CED 3**

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

##### **A-CED 4**

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .

***Reasoning with Equations and Inequalities***

**Solve systems of equations.**

**A-REI 5**

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**A-REI 6**

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Represent and solve equations and inequalities graphically.**

**A-REI 10**

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)

**A-REI 12**

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

## Unit 5 – Linear Modeling, 3 weeks

This unit introduces statistical tools to investigate bivariate data that include scatterplots, and lines that fit the data. Students interpret, describe and summarize the data to make determinations regarding linear and non-linear associations, clustering, outliers, positive, negative and no correlations. The treatment continues by using linear, quadratic and exponential functions and correlation coefficients to make inferences and solve problems in the context of data sets. The students learn to distinguish between correlation and causation.

<b>Big Ideas</b>	<b>Essential Questions</b>
The central organizing ideas and underlying structures of mathematics	
<ul style="list-style-type: none"> <li>Patterns in data can help determine structure in a situation, thus possibly enabling people to make predictions.</li> <li>Although scatter plots and trend lines may reveal a pattern, the relationship of the variables may indicate a correlation, but not causation.</li> </ul>	<ul style="list-style-type: none"> <li>Given a set of bivariate data that appears to have a linear pattern in a scatterplot, how do you find a least squares regression line?</li> <li>What does the rate of change and y-intercept represent in a situation?</li> <li>What is the different between correlation and causation?</li> </ul>

### Common Core State Standards

#### **STATISTICS AND PROBABILITY**

##### ***Interpreting Categorical and Quantitative Data***

##### **Summarize, represent, and interpret data on two categorical and quantitative variables**

###### **S-ID 5**

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

###### **S-ID 6**

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

###### **S-ID 6a**

Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

###### **S-ID 6b**

Informally assess the fit of a function by plotting and analyzing residuals.

##### **Interpret linear models**

###### **S-ID 7**

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

###### **S-ID 8**

Compute (using technology) and interpret the correlation coefficient of a linear fit.

###### **S-ID 9**

Distinguish between correlation and causation.

## Unit 6 – Quadratic Functions, 5 weeks

This unit has students investigate quadratic functions and in one variable  $f(x) = ax^2 + bx + c$  and their applications along with being able to transform different quadratic expressions. See that the product of two linear expressions  $(mx + n)$  and  $(px + q)$  leads to a quadratic function of the form  $f(x) = ax^2 + bx + c$ ; represent quadratic functions symbolically and graphically; know that the graph of a quadratic function is a parabola; see the effect of each of the parameters  $a$ ,  $b$ , and  $c$  on the graph of  $ax^2 + bx + c$ ; express the vertex of the parabolic graph in terms of these parameters; see the graph of any quadratic function  $ax^2 + bx + c$  is a scaled and shifted version of the parabola  $y = x^2$ ; solve max-min problems involving quadratic functions; explore applications of quadratic functions to problems (e.g., applications of quadratic functions in terms of area, motion under gravity, stopping distance of a car, and revenue in terms of demand and price). Additionally, students will see how the different representations of a quadratic expressions are the same.

### Big Ideas

The central organizing ideas and underlying structures of mathematics

- Quadratic functions are characterized by a linear rate of change, so the rate of rate of change (second derivative) of a quadratic function is constant.
- Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or a minimum and that if the zeros of the quadratic are real; they are symmetric about the  $x$ -coordinate of the maximum or minimum.

### Essential Questions

- What are the similarities and differences between quadratic, exponential, and linear functions?
- How do changes in the values of the parameters in a quadratic function change the behavior of the graph?
- What is the relationship between the number of real roots and the graph of a quadratic equation? Why does this relationship exist?
- How can you translate among the vertex, standard, and factored forms of quadratic equations? What is useful about each form?
- What are the characteristics of quadratic change as shown in a graph, function rule, table, or real-world situation?

## Common Core State Standards

### **FUNCTIONS**

#### **Interpreting Functions**

**Interpret functions that arise in applications in terms of the context**

##### **F-IF 4**

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

##### **F-IF 5**

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.

##### **F-IF 6**

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.



## Analyze functions using different representations

### F-IF 7

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

#### F-IF 7a

Graph linear and quadratic functions and show intercepts, maxima, and minima.

### F-IF 8

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

#### F-IF 8a

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

### F-IF 9

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

## Building Functions

### Build a function that models a relationship between two quantities

#### F-BF.1

Write a function that describes a relationship between two quantities.

##### F-BF.1a

Determine an explicit expression, a recursive process, or steps for calculation from a context.

##### F-BF.1b

Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

### Build new functions from existing functions

#### F-BF 3

Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

## Unit 7 – Quadratic Equations, 4 weeks

There are many questions that can be written and answered using quadratic equations and inequalities. Solving these quadratic equations and inequalities requires the ability to select an appropriate method and use it to arrive at solutions. These methods include factoring, completing the square, using a graph, using a table of values, and using the quadratic formula. Connections should be made to the solution to a quadratic equation and the graphical representation.

In working with the different forms, students consider advantages and disadvantages of each. In using the quadratic formula, standard form is necessary, thus the ability to translate between the various forms of a quadratic function is an important skill for students to acquire. In solving quadratic inequalities, knowledge of the graph of the inequality can help students connect to the algebraic solution. Systems can include a linear equation together with a quadratic equation or two to three quadratic equations used simultaneously.

Big Ideas	Essential Questions
The central organizing ideas and underlying structures of mathematics	
<ul style="list-style-type: none"> <li>Solving quadratics can be done through a variety of methods (e.g., graphing, factoring, completing the square, and quadratic formula).</li> </ul>	<ul style="list-style-type: none"> <li>What is the relationship between the number of real roots and the graph of a quadratic equation? Why does this relationship exist?</li> <li>What are the characteristics of quadratic change when using a graph, function rule, table, or real-world situation?</li> <li>How can completing the square be used to prove the quadratic formula?</li> <li>What are some clues that will allow you to choose an appropriate method to solve a quadratic equation; factoring, completing the square, using the quadratic formula, graphing, using a table?</li> <li>What are the possible numbers of solutions for a system of equations where one equation is linear and the other is quadratic?</li> </ul>

### Common Core State Standards

#### ***NUMBER AND QUANTITY***

##### ***The Real Number System***

**Use properties of rational and irrational numbers.**

##### **N-RN.3**

Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

##### ***The Complex Number System***

**Use complex numbers in polynomial identities and equations.**

##### **N-CN.7**

Solve quadratic equations with real coefficients that have complex solutions.

## **ALGEBRA**

### ***Seeing Structure in Expressions***

#### **Write expressions in equivalent forms to solve problems**

##### **A-SSE 3**

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

##### **A-SSE 3a**

Factor a quadratic expression to reveal the zeros of the function it defines.

##### **A-SSE 3b**

Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

### ***Reasoning with Equations and Inequalities***

#### **Solve equations and inequalities in one variable**

##### **A-REI 4**

Solve quadratic equations in one variable.

##### **A-REI 4a**

Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

##### **A-REI 4b**

Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

#### **Solve systems of equations**

##### **A-REI 7**

Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*

## Unit 8 – Statistics, 2 weeks

This unit builds upon statistics introduced in sixth through Pre-Algebra. The concepts in this unit include understanding statistical variability, interpreting and describing distributions of data and understanding the association in bivariate data. The unit begins with interpreting, summarizing and representing single variable categorical and quantitative data sets using plots, graphs, measures of center and spread. The treatment continues with two variables data sets. Students should become familiar with the properties of the normal curve and measures of variance.

<b>Big Ideas</b>	<b>Essential Questions</b>
The central organizing ideas and underlying structures of mathematics	
<ul style="list-style-type: none"> <li>• Data can be analyzed to reveal patterns and to make inferences.</li> <li>• The normal distribution is a fundamental component of statistical inference.</li> </ul>	<ul style="list-style-type: none"> <li>• How do we make predictions and informed decisions based on current numerical information?</li> <li>• What are the advantages and disadvantages of analyzing data by hand versus by using technology?</li> <li>• What is the potential impact of making a decision from data that contains one or more outliers?</li> </ul>

### Common Core State Standards

#### ***STATISTICS AND PROBABILITY***

##### ***Interpreting Categorical and Quantitative Data***

##### **Summarize, represent, and interpret data on a single count or measurement variable**

###### **S-ID 1**

Represent data with plots on the real number line (dot plots, histograms, and box plots).

###### **S-ID 2**

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

###### **S-ID 3**

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

###### **S-ID 4**

Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.