Mathematics

Fairfield Public Schools

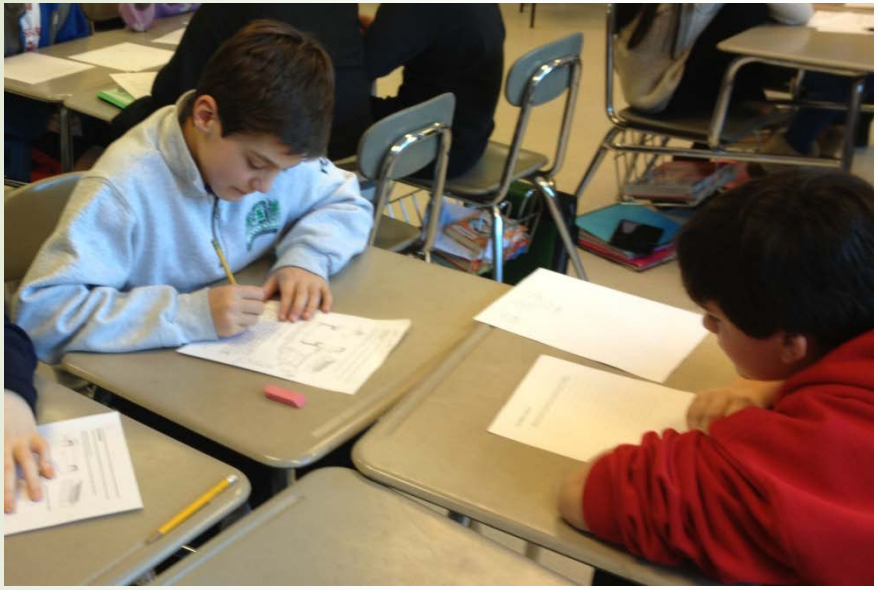
Grade 6 to Geometry

Purpose of Tonight's Presentation

* Overview of the curriculum review process
* Overview of the curriculum document structure
* Examples illustrating improvements to the curriculum
* Illustration of the domains of the CCSS & expected fluencies
* Resource search process & recommendation
* Next steps

* Board of Education Members
* Teachers & Administrators
* Members of the Public

Welcome & Thank You



Curriculum Presentation

Why was the curriculum revised?

- To improve the mathematical learning for all students.
  - Middle School: CMT
  - High School: SAT & CAPT
- Last curriculum revision was 2006
- Connecticut's adoption of the CCSS in July 2010
- SBAC Assessment in 2014-15
- Align to the approved Pre-K to 5 curricula

How was the curriculum developed?

* Research and review
* Organized standards into units of study
* Parent focus group
* Curriculum posted to the district website for community feedback
* Curriculum council review and feedback

Curriculum Document Structure

PRE-ALGEBRA 7

Critical Areas of Focus

In the Pre-Algebra 7 course, instructional time should focus on five critical areas: (1) developing understanding of and applying proportional relationships; (2) drawing inferences about populations based on samples, (3) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (4) grasping the concept of a function and using functions to describe quantitative relationships; (5) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.
2. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.
3. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m·A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.
4. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.
5. Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Pacing Guide

1st Marking Period			2nd Marking Period		3rd Marking Period			4th Marking Period	
September	October	November	December	January	February	March	April	May	June
Unit 1 **Proportional Relationships** 5 weeks	Unit 2 **Algebraic Reasoning II** 4 weeks	Unit 3 **Inferences about Populations** 3 weeks	Unit 4 **Probability** 3 weeks	Unit 5 **Pythagorean Theorem** 4 weeks	Unit 6 **Congruence and Similarity** 3 weeks	Unit 7 **Linear Relationships** 5 weeks		Unit 8 **Systems of Linear Relationships** 3 weeks	Unit 9 **Volume** 3 weeks

Course Overview

Central Understandings

Insights learned from exploring generalizations through the essential questions. (Students will understand that…)

- Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools, and technologies.
- Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
- Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools, and technologies.
- Data can be analyzed to make informed decisions using a variety of strategies, tools, and technologies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable and informed decisions?

Assessments

- Formative Assessments
- Summative Assessments

Grade Seven Standards for Mathematical Practice

The K-12 Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. This page gives examples of what the practice standards look like at the specified grade level. Students are expected to:

Standards	*Explanations and Examples*
1. Make sense of problems and persevere in solving them.	In grade 7, students solve problems involving ratios and rates and discuss how they solved them. Students solve real world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2. Reason abstractly and quantitatively.	In grade 7, students represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In grade 7, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.
4. Model with mathematics.	In grade 7, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students explore covariance and represent two quantities simultaneously. They use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context.
5. Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data. Students might use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms.
6. Attend to precision.	In grade 7, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students define variables, specify units of measure, and label axes accurately. Students use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph. Students apply properties to generate equivalent expressions (i.e. $6 + 2x = 3\ (2 + x)$ by distributive property) and solve equations (i.e. $2c + 3 = 15$, $2c = 12$ by subtraction property of equality), $c=6$ by division property of equality). Students compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume. Students examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities.
8. Look for and express regularity in repeated reasoning.	In grade 7, students use repeated reasoning to understand algorithms and make generalizations about patterns. During multiple opportunities to solve and model problems, they may notice that $a/b \div c/d = ad/bc$ and construct other examples and models that confirm their generalization. They extend their thinking to include complex fractions and rational numbers. Students formally begin to make connections between covariance, rates, and representations showing the relationships between quantities. They create, explain, evaluate, and modify probability models to describe simple and compound events.

Unit 1 – Proportional Relationships, 5 weeks

tudents studied rates and ratios in sixth grade. In this unit, students reason proportionally. They use ratios as a basis of comparison between two sets of ata. They observe related data in the form of a table and look for patterns connecting these data values. Plotting the paired data points to see a graphical epresentation, and writing an equation that shows the relationship of the data in the table further strengthens this understanding. When the change observed the table is constant, students connect to a linear graph. This demonstrates a proportional relationship across multiple representations and deepens the nderstanding of these characteristics. The unit rate studied in grade six is now a focus of rate of change used in writing linear equations in grade seven.

ther concepts in this unit include solving problems to find an unknown part of a proportion and applying proportional reasoning to real-world contexts. tudents think proportionally in such situations as calculating sales tax, interest, and commissions; scale drawings; and unit pricing.

Big Ideas

- Reasoning with ratios involves attending to and coordinating two quantities.
- Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
- Ratios are often used to make "part-to-part" comparisons, but fractions are not.
- Equivalent ratios can be created by iterating and/or partitioning a composed unit.
- A rate is a set of infinitely many equivalent ratios.
- Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.
- A proportion is a relationship of equality between two ratios.

Essential Questions

- How do you find and compare unit rates?
- How can you use tables and equations to identify and describe proportional relationships?
- How can you use graphs to represent and analyze proportional relationships?
- How do you use percents to solve problems?

Common Core State Standards

RATIOS AND PROPORTIONAL RELATIONSHIPS

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ to ¼ miles per hour, equivalently 2 miles per hour.*

7.RP.2

Recognize and represent proportional relationships between quantities.

7.RP.2a

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Curriculum Document in Use

Unit 2 – Numerical and Proportional Reasoning, 9.5 weeks top

Standards

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

Students should understand that a variety of numerical representations can be used to describe quantitative relationships.

2.1a Students should compare and order integers, powers, and roots using number lines and grids.

2.1b Students should extend the understanding of scientific notation to very small numbers.

Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

2.2a Students should solve problems involving fractions, decimals, ratios and percents.

2.2b Students should make generalizations about operations with very large and very small numbers.

2.2c Students should connect the exponential growth and decay models to repeated multiplication by the same factor.

Unit Objective

Students will be able to:

- use numbers and their properties to compute flexibly and fluently and to reasonably estimate measures and quantities.
- use a variety of numerical representations in the base ten system to describe quantitative relationships.

Essential Question

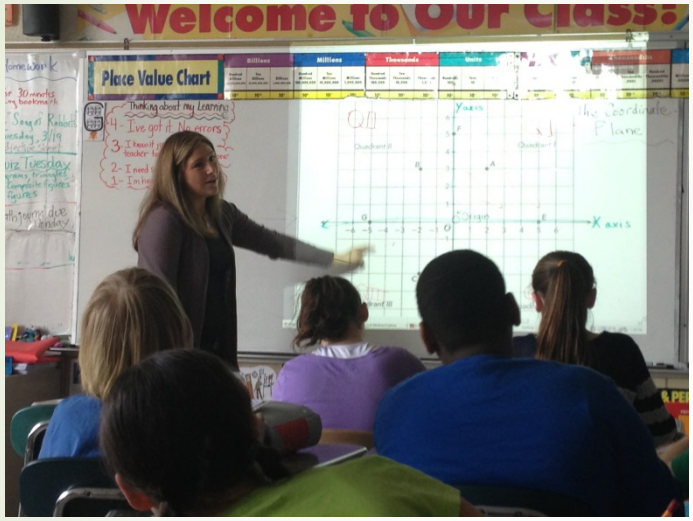
- How are quantitative relationships represented by numbers?

Focus Questions

- How are the base ten number system and fractions, decimals, percents and ratios related?
- How can fractions, decimals, percents and ratios be used to describe and model real world problems?
- How can you locate, label, and order numbers on number lines and scales?
- How do you use the properties of exponents to solve a variety of real world problems?

Assessment

- Payroll Problem



Course Improvements

Similar content:

* Integers and rational numbers
* Area concepts
* Ratios and proportions
* Data and statistics
* Algebraic reasoning

Math 6 Similarities

Math 6 Improvements

Content mastery moved to an earlier grade

* Addition and subtraction of fractions **(Grade 5)**
* Area of squares and rectangles **(Grade 4 & 5)**
* Volume **(Grade 5)**

Content added

* Division of fractions with models and applications
* Use of ratio tables
* Determining area of composite figures
* Using nets to determine surface area
* Absolute value & Inequalities
* Box and whisker plots, quartiles, mean absolute deviation

Similar content:

* **Integers and rational numbers**
* **Area concepts**
* **Ratios and proportions**
* **Data and statistics**
* **Algebraic reasoning**

Transition to Pre-Algebra Similarities

Transition to Pre-Algebra Improvements

Content mastery moved to an earlier grade

* Addition and subtraction of fractions **(Grade 5)**
* Area of squares and rectangles **(Grade 4 & 5)**
* Volume **(Grade 5)**

Content added

* Integers and rational numbers operations
* Two- & three-dimensional Geometry
* In addition to the Math-6 topics noted earlier

Similar content:

* **Integers and rational numbers**
* **Simple equations**
* **Area, perimeter, and volume**
* **Ratios and proportions**

Math 7 Similarities

Math 7 Improvements

Content mastery moved to an earlier grade

- Operations with fractions (Grade 5)
- Operations with decimals (Grade 5 & 6)
- Primes, factors, multiples, divisibility (Grade 5 & 6)
- Exponents and powers (Grade 6)

Content added

* 2-step equations and inequalities
* Graph solution sets of inequalities.
* Derivations of formulas
* Cross-sections
* Test for equivalent ratios by graphing
* Mean absolute deviation

Similar content:

* **Solving linear equations**
* **Measures of central tendency**
* **Ratio and proportions**
* **Similarity and congruence**

Pre-Algebra 7 Similarities

Content mastery moved to an earlier grade

* Exponents (Grade 6)
* Powers of 10 (Grade 5)
* Order/compare rational numbers (Grade 6)
* Scatter plots (Grade 6)

Content added

* Solve and graph multi-step inequalities.
* Linear graphs
* Mean absolute variation
* Prove and apply the Pythagorean Theorem
* Dilations of figures
* Types of angles for parallel lines cut by a transversal.
* Analyzing non-linear graphs.
* System of equations

Pre-Algebra 7 Improvements

Similar content:

* Real Numbers
* Solving linear equations
* Statistics and probability
* Pythagorean Theorem
* Transformations

Pre-Algebra 8 Similarities

Pre-Algebra 8 Improvements

Content mastery moved to an earlier grade

* Order of operations (Grade 6)
* Computation with rational numbers (Grade 7)
* Equivalence of fractions, decimals, percents and Ratios (Grade 7)
* Number lines and grids (Grade 6)
* Basic coordinate plane (Grade 6)
* Volume of cubes and right prisms (Grade 7)

Content added

* Extension of the Pythagorean Theorem
* Functions
* Slope (slope-intercept form)
* Systems of linear relationships
* Laws of exponents

Algebra 8 Similarities

Similar content:

- **Graphing linear equations**
- **Solving equations**
- **Systems of equations**
- **Linear modeling**
- **Quadratic functions and equations**

Algebra 8 Improvements

Content mastery moved to an earlier grade

* Algebraic expressions and equations (grade 6 & 7)
* Basic graphing (grade 6)

Content added

* Real numbers
* Family of functions
* Correlation vs. causation
* Quadratics – completing the square
* Statistics

Algebra 12 Similarities

Similar content:

- **Linear functions**
- **Solving one and two-variable equations**
- **Graphing equations**
- **Functions and relations**
- **Systems of equations**
- **Factoring**
- **Quadratics**

Content mastery moved to an earlier grade

* Laws of exponents (grade 8)
* Algebraic expressions (grade 6 & 7)

Content added

* Statistics
  * Standard deviation
  * Correlation vs. causation
* Family of functions
* Quadratics - completing the square

Algebra 12 Improvements

Similar content:

* Geometric definitions & transformations
* Triangle congruence
* Properties of quadrilaterals
* Properties circles
* Trigonometry

Geometry 21/22 Similarities

Geometry 21/22 Improvements

Content mastery moved to an earlier grade

- Shapes in space (Grade 6)
  - Isometric drawings and orthogonal views
  - Spatial relationships

Content added

- Additional work with constructions and transformations
- Increased focus on conics
  - Equation of parabola given focus and directrix
  - Completing the square to find center and radius of a circle
- Law of Sines and Cosines
- Finding the area of a triangle using A=1/2ab(sin C)

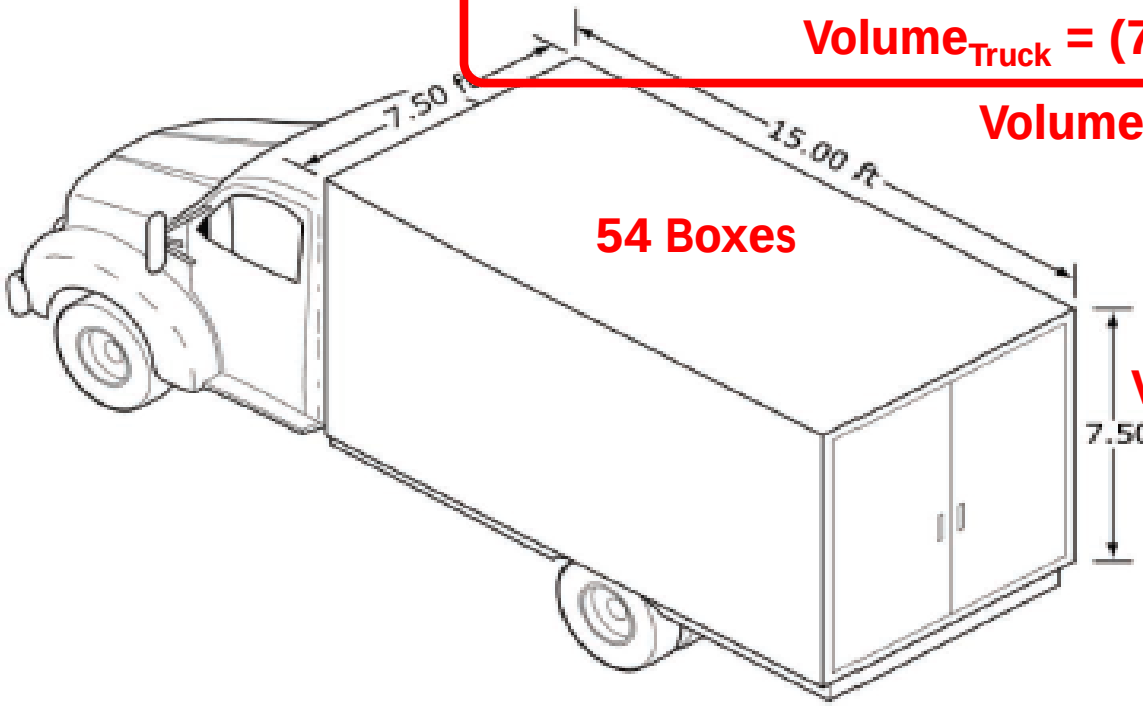
Grade	K	1	2	3	4	5	6	7	8	HS Conceptual Categories
Domains	Counting & Cardinality						Ratios & Proportional Relationships		Functions	Functions
	Operations and Algebraic Thinking						Expression and Equations			Algebra
	Number and Operations in Base Ten						The Number System			Number & Quantity
				Fractions						
	Measurement and Data						Statistics and Probability			Statistics & Probability
	Geometry						Geometry			Geometry

Cube-shaped boxes will be loaded into the cargo hold of a truck. The cargo hold of the truck is in the shape of a rectangular prism. The edges of each box measure 2.50 feet and the dimensions of the cargo hold are 7.50 feet by 15.00 feet by 7.50 feet, as shown below.

Volume = LWH

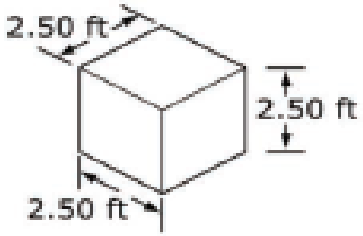
$Volume_{Truck}$ = (7.5 ft)(15 ft)(7.5 ft)

$Volume_{Truck}$ = 843.75 ft^3



54 Boxes

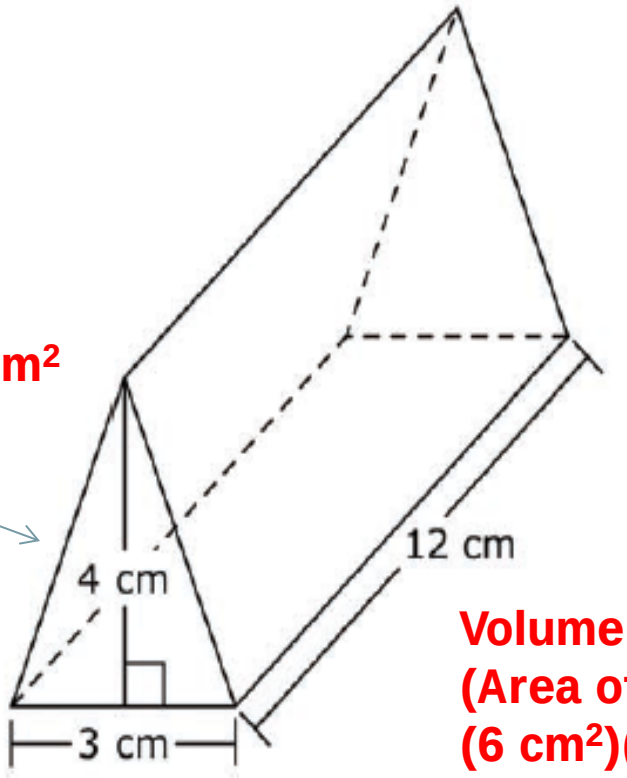
$Volume_{Box}$ = 15.625 ft^3



What is the volume, in cubic feet, of each box?

Determine the number of boxes that will completely fill the cargo hold of the truck. Use words and/or numbers to show how you determined your answer.

Look at the triangular prism below. Each triangular face of the prism has a base of 3 centimeters (cm) and a height of 4 cm. The length of the prism is 12 cm.



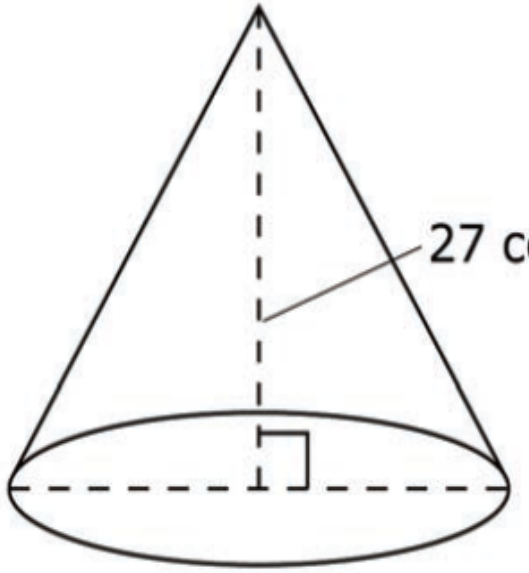
Area of the Base = 6 cm^2

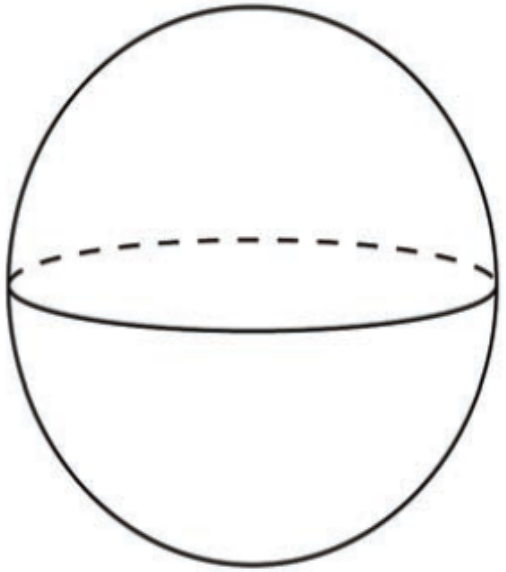
Volume =
(Area of the base)(height) =
(6 cm^2)(12 cm) = 72 cm^3

What is the volume, in cm^3, of this triangular prism?

This cone and sphere have equal volumes.

Volume of the Cone ≈ 7238 cm³





$7238 \text{ cm}^3 = \frac{4}{3}\pi r^3$

not drawn to scale

radius = 12 cm

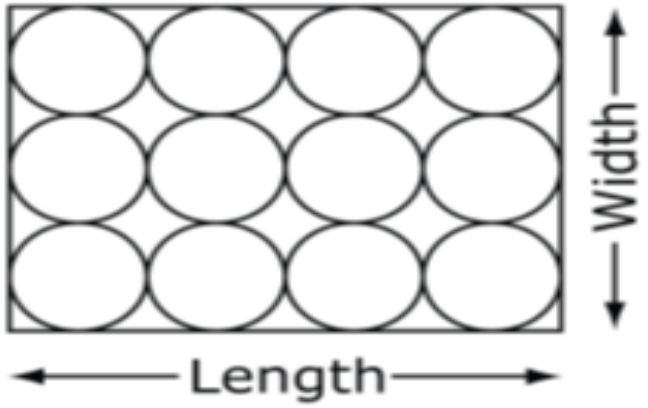
What is the radius of the sphere? ☐ Centimeters

Packaging Cans

You have been asked to be a consultant for a beverage company. The company president would like you to investigate how soda cans are packaged. Cans are constructed in such a way that they are not truly cylinders, but for the purpose of your investigation, we will assume that they are right circular cylinders.

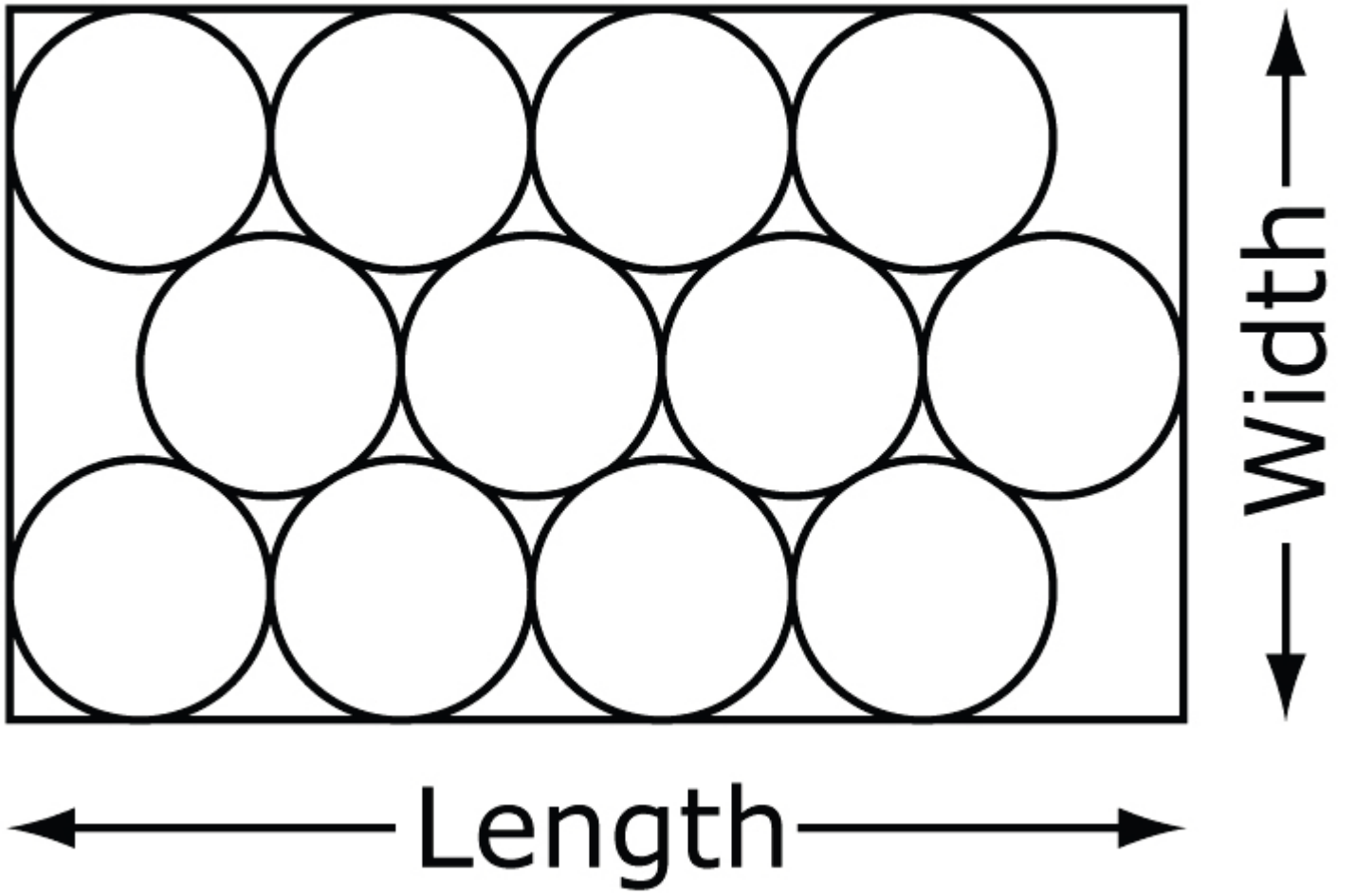
The current boxes used to package soda cans have rectangular bases. The 12 cans in a box are stacked in one layer. The diagram below shows Stacking Method A, a 3-can by 4-can arrangement.

Stacking Method A



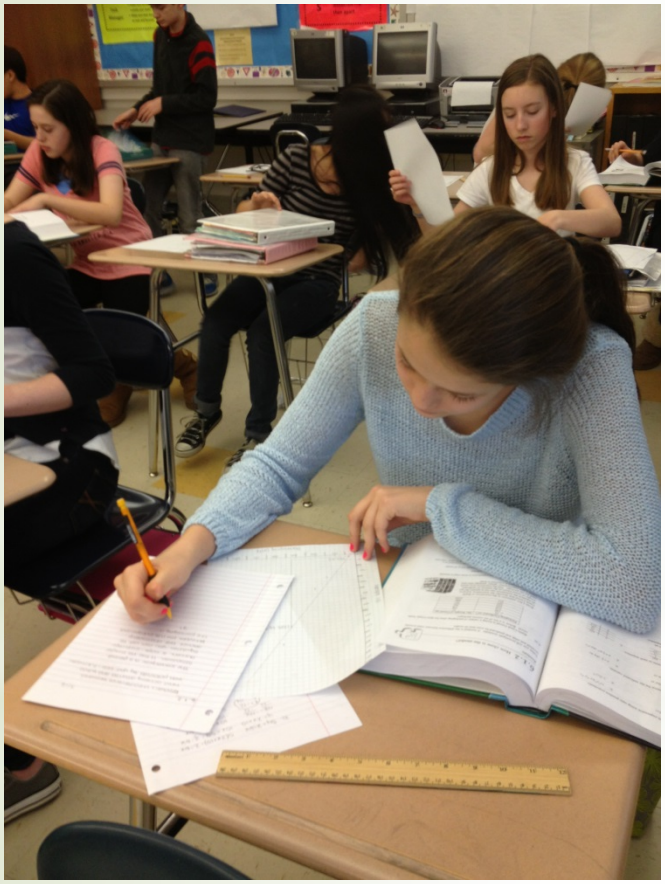
Find all possible one-layer stacking arrangements for 12 cans in a rectangular box where the cans touch as shown. Show them in the space below. The number of cans along the length and the width must be factors of 12.

Stacking Method B



*Fluencies in the Middle Grades into High School

Grade	Fluency
K	Add and subtract within 5
1st	Add and subtract within 10
2nd	Add and subtract multi-digit numbers
3rd	Compute with all four operations
4th	Compute multi-digit whole numbers using efficient strategies with all 4 operations
5th	Compute with fractions and decimals
6th	**Multi-digit division** **Multi-digit decimal operations**
7th	**Solve ax + b = c and** **a(x + b) = c**
8th	**Solve system of equations by Inspection**
Algebra	**Solve complex equations and inequalities** **Factor quadratics**



Resource Search Process

Resource Review Process

* Initial teacher search committee
* Final resource review committee
* Teacher survey
* Community viewing and feedback

Middle School Resource

Resource	Instructional Support (4 Attributes)	Differentiation (3 Attributes)	Assessment/Data Analysis (1 Attribute)	Skill Practice (3 Attributes)	Overall (All 13 Attributes)
Glencoe Math	3.00	3.43	2.94	3.12	3.16
Math in Focus	2.36	2.85	2.61	2.39	2.62
Big Ideas	3.25	3.35	3.01	3.11	3.22

Curriculum	Big Ideas	Glencoe Math
Math 6	3.71	3.50
Transition to Pre-Algebra	3.56	2.94
Math 7	3.57	3.00
Pre-Algebra 7	3.61*	3.44
Pre-Algebra 8	3.57	3.71
Overall Alignment Rating	**3.60**	**3.32**

*Updated from newly released Advanced 2 Big Ideas Learning Resource

Resource Data: Alignment

Big Ideas Learning

Middle School Resource Recommendation

Algebra and Geometry Resources

Resource	Instructional Support (4 Attributes)	Differentiation (3 Attributes)	Assessment/Data Analysis (1 Attribute)	Skill Practice (3 Attributes)	Overall (All 13 Attributes)
Discovering Algebra/Geometry	3.32	2.67	2.82	3.07	**3.05**
Pearson Common Core	2.64	2.95	3.00	3.26	**3.03**
College Preparatory Mathematics Core Connections	3.29	2.60	2.46	2.60	**2.83**

Additional CPM Analysis

* The distribution of grades in 8th grade Algebra has improved when compared to prior years in both the first and second quarters.
* The distribution of grades in the high school in Algebra is consistent with the 5 year average.
* Student individual performance on the midterm at the high school was statistically the same on identical problems.
* The average grade on the 8th grade Algebra midterm was an 87%.

Additional CPM Analysis

* **Student and Parent Survey Results**
  * 51% of students believe working in small groups helps them stay engaged.
  * 72% of the students believe the teacher provides feedback to the individual or groups during the lesson.
  * 46% of parents surveyed believed their student's confidence in math improved.
  * 34% of parents surveyed believed that the online homework help did supply enough support.

Curriculum	Pearson Common Core Algebra & Geometry	Discovering Algebra & Geometry
Algebra 8	2.94	3.11
Algebra 12	3.13	2.50
Geometry (both 22 and 21)	3.75	3.19
Overall Alignment Rating	**3.27**	**2.93**

Resource Data: Alignment

Pearson Common Core

Algebra & Geometry Resource Recommendation

Next Steps

Next Steps: Curriculum Implementation

* Curriculum implementation guides
* Common assessments from the curriculum
* Professional development

Next Steps: Ongoing Evaluation

* Common assessments
  - -internal
  - -external

* Classroom observations

Anticipated Costs

* 3775 Students across nine different courses
* **Middle School Costs:**
  * $171,000 for the Big Ideas Resources
* **High School Costs:**
  * $109,000 for the Pearson Resources
* **Summer Work – Implementation Account**
  * $19,400
* **Approximately Cost to District: $294,000**
  * Costs per student is $74 per student for resources

Thank You and Questions