Curriculum Development In the Fairfield Public Schools

FAIRFIELD PUBLIC SCHOOLS FAIRFIELD, CONNECTICUT

## ADVANCED ALGEBRA/GEOMETRY 42

## ADVANCED ALGEBRA/GEOMETRY 42

## Statement of Purpose

Advanced Algebra/Geometry 42 is a senior course that is an extension of the algebra and geometry programs. It is designed to prepare students for success in college, their careers, and daily lives in the $21^{\text {st }}$ century. This course will help students develop their abilities to explore, solve and analyze mathematical situations. They will apply concepts while thinking critically, working cooperatively and communicating their ideas clearly.

This course is built on the idea that students develop better conceptual understanding of mathematics and stronger problem solving skills when they see the connections among different branches of mathematics, are actively involved in the learning process, and continually build on prior knowledge.

## Audience

The course is intended for college bound seniors who have successfully completed Algebra 32 and want to reinforce and expand concepts of algebra and geometry or students with B or better in Integrated Algebra/Geometry 33.

## Prerequisites

Successful completion of Algebra 32 or a B or better in Integrated Algebra/Geometry 33, and teacher recommendation.

## Course Description

Advanced Algebra/Geometry 42 is an extension of topics in algebra, geometry, and data analysis. SAT problems and questions will be integrated into the first semester. Full year topics include: functions, analyzing data, linear programming, sequences and series, and trigonometry which will prepare seniors for an introduction to material that may be included in college entrance exams and/or introductory math courses in college. Credit is granted by semester.

## Course Objectives

Students will be able to:

- use trigonometry and geometry to determine angles, arcs and sides of polygons.
- define and utilize the properties of inscribed polygons, inscribed circles and regular and semi-regular polyhedra.
- use the Law of Cosines and the Law of Sines to solve triangles.
- represent, graph and analyze functions and equations.
- model problem situations using systematic lists and systems of equations and inequalities.
- recognize and describe linear, quadratic, polynomial, radical, and rational functions.
- write and use logarithmic and exponential functions to model situations.
- develop, symbolize and classify sequences and series.
- use sigma notation to represent series and evaluate sums.
- determine, define and evaluate geometric and arithmetic sequences and series.
- gather, organize, model, represent and interpret sets of data.
- use technology to explore and analyze data.
- model data in a variety of ways.


## Skill Objectives

Students will:

- investigate formulas for the sums of interior and exterior angle measures in polygons.
- examine the relationships between inscribed and circumscribed polygons and circles.
- use indirect methods including the Pythagorean Theorem, trigonometric ratios and proportions in similar figures to solve a variety of measurement problems.
- use techniques of trigonometry to make indirect measurements of lengths and angles to solve a variety of problems.
- use Pythagorean Theorem to determine the measures of the sides of a triangle.
- apply Triangle Inequality Theorem.
- use law of sines to solve problem situations.
- categorize a graph by type: periodic, damping, saturating, constant.
- compare the rates of change of graphs.
- determine the period and amplitude of a function from its graph.
- write an equation of the horizontal axis of graph.
- sketch a periodic graph using the amplitude period and equation of axis.
- solve systems of two or three linear equations using algebraic methods, graphical methods or matrices.
- use systems of inequalities to model situations and find maximum and minimum values.
- identify the characteristics of functions and relations including domain and range.
- write and interpret functional notation.
- determine from a graph whether an equation represents a function and is either one to one or many to one.
- model and solve problems with linear and linear inequalities.
- determine the intercepts and maximum or minimum values from an equation.
- determine equivalent representations of an algebraic equation or inequality to simplify and solve problems.
- evaluate and interpret the graphs of linear, and polynomial functions.
- determine if a function is quadratic from a graph and an equation.
- solve problem situations using the formula for height or distance.
- describe and compare properties and classes of linear, direct, quadratic, radical, exponential and polynomial.
- write and solve problems involving direct variation.
- identify an appropriate symbolic representation for a function or relation displayed graphically or verbally.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- understand and use optimization strategies including linear programming.
- determine the degree and zeros of a polynomial function.
- simplify and solve radical equations.
- use the discriminant to find the number of real solutions of a quadratic equation.
- perform operations of Complex Numbers.
- use exponential growth and decay functions to model situations.
- rewrite exponential functions in the form $\mathrm{y}=\mathrm{ab}^{\mathrm{x}}$ where x is either positive or negative.
- model situations using exponential functions with base e.
- write expressions with fractional exponents in radical form.
- write radical expressions using a fractional exponent.
- simplify radicals using fractional exponents.
- write an equation in base 10 logarithmic form.
- write an equation in exponential form.
- evaluate base 10 logarithmic expressions.
- solve problems involving financial applications including compound interest.
- identify, describe, create, and generalize numeric patterns with graphs, words and symbolic rules.
- write the next terms of a sequence.
- graph the sequence and apply the concepts of limits to sequences.
- determine the terms of a sequence or series using an explicit formula and recursive formula.
- write the explicit formula and recursive formula for a sequence or a series.
- identify a common difference in an arithmetic sequence.
- identify a common ratio in a geometric sequence.
- classify a sequence or series as arithmetic, geometric or neither.
- use a formula to determine the sum of a finite arithmetic series, a finite geometric series and infinite series.
- write a series in sigma notation.
- evaluate the sum of a sigma notation.
- determine the partial sums of an infinite series.
- collect real data and create meaningful graphical representations of the data.
- develop, use, and explain applications and limitations of linear and non-linear models and regression in a variety of contexts.
- create, use and interpret frequency distribution, least squares, quadratic regressions, box and whisker plots, scatter plots, histograms and relative frequency histograms.
- estimate an unknown value between data points on a graph (interpolation) and make predictions by extending the graph (extrapolation).
- explore and analyze types of frequency distributions.
- identify, describe, create and generalize statistical patterns with graphs and symbolic rules.
- determine and use measures of spread and central tendency to describe and compare sets of data.
- use statistics to compare two sets of data.
- explore the characteristics and applications of the normal distribution and standardized scores.
- apply regression models for bivariate data and use them to formulate predictions.
- interpret the correlation coefficient.


## Math Standards

Algebraic Reasoning: Patterns And Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
1.1 Students should understand and describe patterns and functional relationships. Core
1.1a Students should describe relationships and make generalizations about patterns and functions.
Extended
1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.
1.2 Students should represent and analyze quantitative relationships in a variety of ways. Core
1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.
Extended
1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.
1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.
Core
1.3a Students should manipulate equations, inequalities, and functions to solve problems.

Extended
1.3a Students should use and extend algebraic concepts to include real and complex numbers and matrices.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships.
Extended
2.1a Students should extend the understanding of number to include the set of complex numbers. 2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.
Extended
2.2a Students should investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement - Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

## Core

3.3a Students should solve a variety of problems involving one- two- and three-dimensional measurements using geometric relationships and trigonometric ratios.
Extended
3.3a Students should approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Working with Data: Probability and Statistics - Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.
4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.
Core
4.1a Students should create the appropriate visual or graphical representation of real data. Extended
4.1a Students should model real data graphically using appropriate tools, technology and strategies.
4.2 Students should analyze data sets to form hypotheses and make predictions. Core
4.2a Students should analyze real world problems using statistical techniques.
4.3 Students should understand and apply basic concepts of probability.

Extended
4.3b Students should make statistical inferences through the use of probability.

## Information and Technology Standards (to be added)

## Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?
- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?


## Focus Questions

- How do geometric concepts get tested on SAT and placement tests for colleges?
- What are the properties of inscribed polygons?
- How do the Law of Cosines, Law of Sines, trigonometric ratios and Pythagorean theorem lead to the solutions of triangles in problem situations?
- How are the behavior of functions classified and described?
- How do the concepts of functions and equations get tested on SAT and placement tests for colleges?
- What are the differences between linear, quadratic, polynomial, radical, rational, exponential, logarithmic and composite functions?
- How can linear and exponential functions be used to model situations?
- How can the process of linear programming be used to determine an optimal solution?
- How does the concept of numeric patterns get tested on SAT and placement tests for colleges?
- How can geometric and arithmetic sequences and series be used to model situations?
- How can you use sigma to represent a series?
- How can you determine the sum of a series?
- How can you use explicit and recursive formulas to represent terms in sequences or series?
- How does analyzing data get tested on SAT and placement tests for colleges?
- How can a frequency distribution be determined from a model data set?
- How can data be modeled appropriately to form a linear or quadratic equation in order to analyze and make predictions?


## UNITS OF STUDY

## Unit 1: Geometry and Trigonometry

## Math Standards

Geometry and Measurement - Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.
Core
3.3a Students should solve a variety of problems involving one- two- and three-dimensional measurements using geometric relationships and trigonometric ratios.
Extended
3.3a Students should approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

## Essential Question

- How do geometric relationships and measurements help us to solve problems and make sense of our world?


## Focus Questions

- How do geometric concepts get tested on SAT and placement tests for colleges?
- What are the properties of inscribed polygons?
- How do the Law of Cosines, Law of Sines, trigonometric ratios and Pythagorean theorem lead to the solutions of triangles in problem situations?
- How are the behavior of functions classified and described?


## Core Topics

- Review of basic geometry formulas, including pythagorean theorem and trigonometric ratios
- Angles and polygons
- Inscribed polygons
- Law of Cosines
- Law of Sines
- Describing the behavior of functions
- Periodic function


## Unit Objectives

Students will be able to:

- use trigonometry and geometry to determine angles, arcs and sides of polygons.
- define and utilize the properties of inscribed polygons, inscribed circles and regular and semi-regular polyhedra.
- use the Law of Cosines and the Law of Sines to solve triangles.


## Skill Objectives

## Students will:

- investigate formulas for the sums of interior and exterior angle measures in polygons.
- examine the relationships between inscribed and circumscribed polygons and circles.
- use indirect methods including the Pythagorean Theorem, trigonometric ratios and proportions in similar figures to solve a variety of measurement problems.
- use techniques of trigonometry to make indirect measurements of lengths and angles to solve a variety of problems.
- use Pythagorean Theorem to determine the measures of the sides of a triangle.
- apply Triangle Inequality Theorem.
- use law of sines to solve problem situations.
- categorize a graph by type: periodic, damping, saturating, constant.
- compare the rates of change of graphs.
- determine the period and amplitude of a function from its graph.
- write an equation of the horizontal axis of graph.
- sketch a periodic graph using the amplitude period and equation of axis.


## Sample Assessment

To Be Created

## Pacing

8 weeks

## Unit 2: Functions and Equations

## Math Standards

Algebraic Reasoning: Patterns And Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
1.1 Students should understand and describe patterns and functional relationships. Core
1.1a Students should describe relationships and make generalizations about patterns and functions.

## Extended

1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.
1.2 Students should represent and analyze quantitative relationships in a variety of ways. Core
1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.
Extended
1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.
1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.
Core
1.3a Students should manipulate equations, inequalities, and functions to solve problems.

## Extended

1.3a Students should use and extend algebraic concepts to include real and complex numbers and matrices.

Numerical and Proportional Reasoning - Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
2.1 Students should understand that a variety of numerical representations can be used to describe quantitative relationships.

## Extended

2.1a Students should extend the understanding of number to include the set of complex numbers. 2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

## Extended

2.2a Students should investigate mathematical properties and operations related to objects that are not numbers.

## Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?


## Focus Questions

- How do the concepts of functions and equations get tested on SAT and placement tests for colleges?
- What are the differences between linear, quadratic, polynomial, radical, rational, exponential, logarithmic and composite functions?
- How can linear and exponential functions be used to model situations?
- How can the process of linear programming be used to determine an optimal solution?


## Core Topics

- Recognize and Describe Functions
- Linear Functions
- Quadratic Functions
- Polynomial Functions
- Radical Functions
- Complex Numbers
- Exponential Growth and Decay
- Fractional Exponents
- Logarithmic Functions
- Maxima and Minima
- Systems of Equations and Inequalities
- Linear Programming


## Unit Objectives

Students will be able to:

- represent, graph and analyze functions and equations.
- model problem situations using systematic lists and systems of equations and inequalities.
- recognize and describe linear, quadratic, polynomial, radical, and rational functions.
- write and use logarithmic and exponential functions to model situations.


## Skill Objectives

## Students will:

- solve systems of two or three linear equations using algebraic methods, graphical methods or matrices.
- use systems of inequalities to model situations and find maximum and minimum values.
- identify the characteristics of functions and relations including domain and range.
- write and interpret functional notation.
- determine from a graph whether an equation represents a function and is either one to one or many to one.
- model and solve problems with linear and linear inequalities.
- determine the intercepts and maximum or minimum values from an equation.
- determine equivalent representations of an algebraic equation or inequality to simplify and solve problems.
- evaluate and interpret the graphs of linear, and polynomial functions.
- determine if a function is quadratic from a graph and an equation.
- solve problem situations using the formula for height or distance.
- describe and compare properties and classes of linear, direct, quadratic, radical, exponential and polynomial.
- write and solve problems involving direct variation.
- identify an appropriate symbolic representation for a function or relation displayed graphically or verbally.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- understand and use optimization strategies including linear programming.
- determine the degree and zeros of a polynomial function.
- simplify and solve radical equations.
- use the discriminant to find the number of real solutions of a quadratic equation.
- perform operations of Complex Numbers.
- use exponential growth and decay functions to model situations.
- rewrite exponential functions in the form $\mathrm{y}=\mathrm{ab}^{\mathrm{x}}$ where x is either positive or negative.
- model situations using exponential functions with base e.
- write expressions with fractional exponents in radical form.
- write radical expressions using a fractional exponent.
- simplify radicals using fractional exponents.
- write an equation in base 10 logarithmic form.
- write an equation in exponential form.
- evaluate base 10 logarithmic expressions.
- solve problems involving financial applications including compound interest.


## Sample Assessment

To Be Created

## Pacing

12 weeks

## Unit 3: Sequences and Series

## Math Standards

Algebraic Reasoning: Patterns And Functions - Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
1.1 Students should understand and describe patterns and functional relationships. Core
1.1a Students should describe relationships and make generalizations about patterns and functions.
Extended
1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.
1.2 Students should represent and analyze quantitative relationships in a variety of ways. Core
1.2a Students should represent and analyze linear and non-linear functions and relations symbolically and with tables and graphs.
1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.
Core
1.3a Students should manipulate equations, inequalities, and functions to solve problems.

## Essential Question

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?


## Focus Questions

- How does the concept of numeric patterns get tested on SAT and placement tests for colleges?
- How can geometric and arithmetic sequences and series be used to model situations?
- How can you use sigma to represent a series?
- How can you determine the sum of a series?
- How can you use explicit and recursive formulas to represent terms in sequences or series?


## Core Topics

- Exploring Patterns
- Sequences
- Using Explicit and Recursive Formulas
- Arithmetic and Geometric Sequences
- Arithmetic, Geometric and Infinite Series
- Sigma Notation


## Unit Objectives

Students will be able to:

- develop, symbolize and classify sequences and series.
- use sigma notation to represent series and evaluate sums.
- determine, define and evaluate geometric and arithmetic sequences and series.


## Skill Objectives

Students will:

- identify, describe, create, and generalize numeric patterns with graphs, words and symbolic rules.
- write the next terms of a sequence.
- graph the sequence and apply the concepts of limits to sequences.
- determine the terms of a sequence or series using an explicit formula and recursive formula.
- write the explicit formula and recursive formula for a sequence or a series.
- identify a common difference in an arithmetic sequence.
- identify a common ratio in a geometric sequence.
- classify a sequence or series as arithmetic, geometric or neither.
- use a formula to determine the sum of a finite arithmetic series, a finite geometric series and infinite series.
- write a series in sigma notation.
- evaluate the sum of a sigma notation.
- determine the partial sums of an infinite series.


## Sample Assessment

To Be Created

## Pacing

4 weeks

## Unit 4: Modeling and Analyzing Data

## Math Standards

Working with Data: Probability and Statistics - Data can be analyzed to make informed decisions using a variety of strategies, tools and technologies.
4.1 Students should collect, organize and display data using appropriate statistical and graphical methods.
Core
4.1a Students should create the appropriate visual or graphical representation of real data. Extended
4.1a Students should model real data graphically using appropriate tools, technology and strategies.
4.2 Students should analyze data sets to form hypotheses and make predictions.

Core
4.2a Students should analyze real world problems using statistical techniques.
4.3 Students should understand and apply basic concepts of probability.

Extended
4.3b Students should make statistical inferences through the use of probability.

## Essential Question

- How can collecting, organizing and displaying data help us analyze information and make reasonable predictions and informed decisions?


## Focus Questions

- How does analyzing data get tested on SAT and placement tests for colleges?
- How can a frequency distribution be determined from a model data set?
- How can data be modeled appropriately to form a linear or quadratic equation in order to analyze and make predictions?


## Core Topics

- Methods to display data
- Distributions of Data
- Standard Deviation
- Normal Distributions
- Fitting Linear Models to Data
- Fitting Quadratic Models to Data
- Simulation and Random Numbers


## Unit Objectives

Students will be able to:

- gather, organize, model, represent and interpret sets of data.
- use technology to explore and analyze data.
- model data in a variety of ways.


## Skill Objectives

## Students will:

- collect real data and create meaningful graphical representations of the data.
- develop, use, and explain applications and limitations of linear and non-linear models and regression in a variety of contexts.
- create, use and interpret frequency distribution, least squares, quadratic regressions, box and whisker plots, scatter plots, histograms and relative frequency histograms.
- estimate an unknown value between data points on a graph (interpolation) and make predictions by extending the graph (extrapolation).
- explore and analyze types of frequency distributions.
- identify, describe, create and generalize statistical patterns with graphs and symbolic rules.
- determine and use measures of spread and central tendency to describe and compare sets of data.
- use statistics to compare two sets of data.
- explore the characteristics and applications of the normal distribution and standardized scores.
- apply regression models for bivariate data and use them to formulate predictions.
- interpret the correlation coefficient.


## Sample Assessment

To Be Created

## Pacing

5 weeks

