Curriculum Development
In the Fairfield Public Schools

FAIRFIELD PUBLIC SCHOOLS
FAIRFIELD, CONNECTICUT

ADVANCED PLACEMENT CALCULUS BC

Board of Education Approved 03/27/2007

ADVANCED PLACEMENT CALCULUS BC

Statement of Purpose

Advanced Placement Calculus BC is a calculus course with accelerated pacing for our most capable and committed students with content that extends beyond the AB Calculus program. The course emphasizes a multi-representational approach, with concepts, results, and problems being expressed geometrically, numerically, analytically, and verbally. Technology is used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, and to assist in interpreting results.
Through the Advanced Placement exam or the University of Connecticut Early College Experience (ECE) Program, students may receive up to 8 undergraduate credits of college calculus.

Audience

AP Calculus BC is intended for students who have completed Precalculus 41 with exceptional success. It is expected that students taking this course will take the Advanced Placement Exam.

Prerequisites

AP Calculus BC requires completion of Precalculus 41 with a grade of A or better and teacher recommendation.

Course Description

Advanced Placement Calculus BC consists of a full year of college calculus.
This course is intended for students who have demonstrated exceptional ability and achievement in mathematics, and have successfully completed an accelerated program. To be successful, students must be motivated learners who have mathematical intuition, a solid background in the topics studied in previous courses and the persistence to grapple with complex problems.

Students in the course are expected to take the Advanced Placement exam in May, at a fee, for credit and/or placement consideration by those colleges which accept AP credit. In addition, by virtue of our affiliation with the University of Connecticut's ECE Program, students can apply for 8 college credits for Math 115Q and Math 116Q at the University of Connecticut.
Included in the course of study will be:

- Functions, graphs and limits
- Differential calculus (the derivative and its applications)
- Integral calculus (antiderivatives and their applications)
- Polynomial Approximations and Series

Course Objectives

Students will be able to:

- operate with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- demonstrate an understanding of the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- demonstrate an understanding of the definite integral both as a limit of Riemann sums and as the net accumulation of a rate of change and should be able to use integrals to solve a variety of problems.
- demonstrate an understanding of the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurements.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Skill Objectives

Students will:

- with the aid of technology, predict and explain the observed local and global behavior of a function.
- using analytical information from geometry and calculus, predict and explain the observed local and global behavior of a function.
- explain intuitively the meaning of limit and show it graphically.
- calculate limits using algebra.
- estimate limits from graphs or tables of data.
- explain horizontal and vertical asymptotes in terms of graphical behavior.
- describe asymptotes in terms of limits involving infinity.
- compare relative magnitudes of functions and their rates of change (e.g. contrast exponential vs. polynomial vs. logarithmic growth).
- provide an intuitive explanation of continuity and one based on limits.
- determine continuity of a function at a point based on geometric representation and the definition of continuity.
- describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic and trigonometric.
- analyze essential relations in a problem to determine possible functions that could model the situation.

- solve problems involving exponential growth.
- solve problems involving direct and inverse variation.
- understand and use optimization strategies, including linear programming.
- apply the concepts of limits to sequences and asymptotic behavior of functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- recognize that the slope of the tangent line to a curve represents the rate of change.
- combine, compose and invert functions.
- use logarithms to solve problems.
- perform operations with logarithms.
- recognize the relationships between a conditional statement and its converse.
- test the validity of logical arguments.
- visualize three-dimensional objects from different perspectives and analyze cross-sections, surface area and volume.
- use Cartesian systems to represent, analyze and solve geometric and measurement problems.
- use successive approximation, upper and lower bounds, and limits to solve measurement problems.
- use properties of similarity and techniques of trigonometry to make indirect measurements of lengths and angles to solve a variety of problems.
- explain the derivative as an instantaneous rate of change by using limits and showing it geometrically as the slope of the tangent.
- find derivatives as the limit of the difference quotient (the formal definition of derivative).
- explain the relationship between differentiability and continuity.
- find the slope of a curve at a point. (Including what happens at point where there are vertical tangents and those where there are no tangents).
- find the equation of the tangent or normal line to a curve at a point and find a local linear approximation.
- demonstrate instantaneous rate of change as the limit of average rate of change.
- approximate rate of change from graphs and tables of values.
- recognize and determine the corresponding characteristics of the graphs of f and f'.
- show the relationship between the increasing and decreasing behavior of f and the sign of f'.
- explain the Mean Value Theorem and its geometric consequences.
- translate verbal descriptions into equations involving derivatives and vice versa.
- relate and recognize the corresponding characteristics of the graphs of f, f' and f''.
- describe and demonstrate the relationship between the concavity of f and the sign of f''.

- determine points of inflection and show that they are places where concavity changes.
- analyze curves (including notions of monotonicity and concavity).
- analyze planar curves given in vector form including velocity and acceleration vectors.
- solve equations and situations for optimization, for both absolute (global) and relative (local) extrema.
- model problems involving rates of change, including related rates problems.
- use implicit differentiation for implicitly stated relations, functions and inverse functions.
- interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- explain the geometric interpretation of differential equations via slope fields and the relationship between slope fields and the derivatives of implicitly defined functions.
- determine the numerical solution of differential equations using Euler's Method.
- explain L'Hospital's Rule and its use in determining convergence of improper integrals and series.
- determine the derivatives of functions, including polynomial, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- determine derivatives using the rules for sums, products, and quotients.
- determine derivatives using the chain rule and implicit differentiation.
- determine derivatives of parametric, polar and vector functions.
- describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic and trigonometric.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- solve problems involving exponential growth, including logistic growth models.
- solve problems involving direct and inverse variation.
- understand and use optimization strategies, including linear programming.
- apply the concepts of limits to sequences and asymptotic behavior of functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- recognize that the slope of the tangent line to a curve represents the rate of change.
- use logarithms and vectors to solve problems.
- perform operations with logarithms.
- recognize the relationships between a conditional statement and its converse.
- test the validity of logical arguments.
- visualize three-dimensional objects from different perspectives and analyze cross-sections, surface area and volume.

- use Cartesian and polar systems to represent, analyze and solve geometric and measurement problems.
- examine rotations of plane figures using sketches, coordinates, and function notation to solve related geometric problems.
- use successive approximation, upper and lower bounds, and limits to solve measurement problems.
- use properties of similarity and techniques of trigonometry to make indirect measurements of lengths and angles to solve a variety of problems.
- compute Riemann sums using left, right, and midpoint evaluation points.
- explain and write the definite integral as the limit of Riemann sums over equal subintervals.
- express the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval $\int_a^b f'(x)dx = f(b) - f(a)$.
- use the basic properties of integrals, such as additivity and linearity.
- use Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.
- use integration to determine the area of a region.
- use integration to determine the volume of a solid with known cross section.
- use integration to determine the average value of a function.
- use integration to determine the distance traveled by a particle along a line.
- use integration to determine the length of a curve (including a curve given in parametric form).
  (The emphasis should always be on using the integral of a rate of change to give accumulated change or by setting up an approximating Riemann Sum and representing its limit as a definite integral.)
- use the Fundamental Theorem to evaluate definite integrals.
- use the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.
- determine antiderivatives of functions which follow directly from the derivatives of basic functions.
- determine antiderivatives by substitution of variables (including change of limits for definite integrals), integration by parts, and simple partial fractions (nonrepeating linear factors only).
- solve improper integrals (as limits of definite integrals).
- determine specific antiderivatives using initial conditions, including applications to motion along a line.
- solve separable differential equations and use them in modeling, including $y' = ky$ and exponential growth.
- solve logistic differential equations and use them in modeling population growth.
- define a series as a sequence of partial sums and convergence as the limit of the sequence of partial sums.
- explain series through examples such as: decimal expansion, geometric series, the harmonic series, alternating series with error bound.

- relate the terms of series to the areas of rectangles and their relationship to improper integrals.
- use the integral test, where appropriate, to determine whether a series converges, including its use in testing the convergence of *p*-series.
- use the ratio test for convergence or divergence.
- use the limit comparison test for convergence or divergence.
- use the comparison test for convergence or divergence.
- use the alternating series test to determine if a series is convergent and if it is absolutely convergent.
- explain how Taylor and Maclaurin series approximate a polynomial with graphical demonstration.
- find Maclaurin series and the general Taylor Series centered at x = a.
- write Maclaurin series for the functions e^x, sin *x*, cos *x*, and 1/(1-x).
- manipulate Taylor series and use shortcuts to compute them, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
- write power series to represent appropriate functions.
- determine the radius and interval of convergence of power series.
- use the Lagrange error bound for Taylor Polynomials.
- describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic and trigonometric.
- analyze essential relations in a problem to determine possible functions that could model the situation..
- solve problems involving exponential growth.
- solve problems involving direct and inverse variation.
- understand and use optimization strategies, including linear programming.
- apply the concepts of limits to sequences and asymptotic behavior of functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- recognize that the slope of the tangent line to a curve represents the rate of change.
- combine, compose and invert functions.
- use logarithms to solve problems.
- perform operations with logarithms.
- recognize the relationships between a conditional statement and its converse
- use successive approximation, upper and lower bounds, and limits to solve measurement problems.

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

Extended

1.1 Students should understand and describe patterns and functional relationships.

1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

1.3a Students will use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

Extended

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

2.2a Students will investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

Extended

3.1 Students should use properties and characteristics of two- and three-dimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems.

3.1a Students will use methods of deductive and inductive reasoning to make, test, and validate geometric conjectures.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Information and Technology Standards (to be added)

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?

- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is Calculus and what role does it play as a tool in science, business and other areas of study?
- What is the structure of the Advanced Placement exam? How can students maximize their efforts to be successful on the exam, in addition to having knowledge of the course content?
- How can calculus and the concepts of limit and continuity assist us in analyzing curves?
- What is a derivative, how do we determine it, and what are its applications in the real world?
- What is an integral (definite and indefinite), how can it be determined and/or evaluated and how can it be applied to problems in the real world?
- Given data from a function or relation how can its derivative and/or integral be found or approximated?

UNITS of STUDY

Unit 1: Functions, Graphs, and Limits

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

Extended

1.1 Students should understand and describe patterns and functional relationships.

1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

1.3a Students will use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

Extended

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

2.2a Students will investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

Extended

3.1 Students should use properties and characteristics of two- and three-dimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems.

3.1a Students will use methods of deductive and inductive reasoning to make, test, and validate geometric conjectures.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is calculus and what role does it play as a tool in science, business, and other areas of study?
- What is the structure of the Advanced Placement exam? How can students maximize their efforts to be successful on the exam, in addition to having knowledge of the course content?

Core Topics

- Analyze Graphs
- Limits of a function (including one-sided limits)
- Asymptotic and unbounded behavior
- Continuity as a property of functions

Unit Objectives

Students will be able to:

- operate with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurements.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Skill Objectives

Students will:

- with the aid of technology, predict and explain the observed local and global behavior of a function.
- using analytical information from geometry and calculus, predict and explain the observed local and global behavior of a function.
- explain intuitively the meaning of limit and show it graphically.
- calculate limits using algebra.
- estimate limits from graphs or tables of data.
- explain horizontal and vertical asymptotes in terms of graphical behavior.
- describe asymptotes in terms of limits involving infinity.

- compare relative magnitudes of functions and their rates of change (eg contrast exponential vs polynomial vs logarithmic growth).
- provide an intuitive explanation of continuity and one based on limits.
- determine continuity of a function at a point based on geometric representation and the definition of continuity.
- describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic and trigonometric.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- solve problems involving exponential growth.
- solve problems involving direct and inverse variation.
- understand and use optimization strategies, including linear programming.
- apply the concepts of limits to sequences and asymptotic behavior of functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- recognize that the slope of the tangent line to a curve represents the rate of change.
- combine, compose and invert functions.
- use logarithms to solve problems.
- perform operations with logarithms.
- recognize the relationships between a conditional statement and its converse.
- test the validity of logical arguments.
- visualize three-dimensional objects from different perspectives and analyze cross-sections, surface area and volume.
- use Cartesian systems to represent, analyze and solve geometric and measurement problems.
- use successive approximation, upper and lower bounds, and limits to solve measurement problems.
- use properties of similarity and techniques of trigonometry to make indirect measurements of lengths and angles to solve a variety of problems.

Sample Assessment

Released open-ended and multiple choice questions from past AP exams

Pacing

1 week

Unit 2: Derivatives

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
Extended

1.1 Students should understand and describe patterns and functional relationships.
1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.
1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.
1.3a Students will use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
Extended

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.
2.2a Students will investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
Extended

3.1 Students should use properties and characteristics of two- and three-dimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems.
3.1a Students will use methods of deductive and inductive reasoning to make, test, and validate geometric conjectures.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.
3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.
3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is calculus and what role does it play as a tool in science, business, and other areas of study?
- How can calculus and the concepts of limit and continuity assist us in analyzing curves?
- What is a derivative, how do we determine it, and what are its applications in the real world?
- Given data from a function or relation how can its derivative and/or integral be found or approximated?
- What is the structure of the Advanced Placement exam? How can students maximize their efforts to be successful on the exam, in addition to having knowledge of the course content?

Core Topics

- Concept of the derivative
- Derivative at a point
- Derivative of a function
- Applications of derivatives
- Computation of derivatives

Unit Objectives

Students will be able to:

- operate with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- demonstrate an understanding of the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- model a written description of a physical situation with a function, a differential equation.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurements.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Skill Objectives

Students will:

- explain the derivative as an instantaneous rate of change by using limits and showing it geometrically as the slope of the tangent.
- find derivatives as the limit of the difference quotient (the formal definition of derivative).
- explain the relationship between differentiability and continuity.
- find the slope of a curve at a point. (Including what happens at point where there are vertical tangents and those where there are no tangents).
- find the equation of the tangent or normal line to a curve at a point and find a local linear approximation.
- demonstrate instantaneous rate of change as the limit of average rate of change.
- approximate rate of change from graphs and tables of values.
- recognize and determine the corresponding characteristics of the graphs of f and f'.
- show the relationship between the increasing and decreasing behavior of f and the sign of f'.
- explain the Mean Value Theorem and its geometric consequences.
- translate verbal descriptions into equations involving derivatives and vice versa.
- relate and recognize the corresponding characteristics of the graphs of f, f', and f''.
- describe and demonstrate the relationship between the concavity of f and the sign of f''.
- determine points of inflection and show that they are places where concavity changes.
- analyze curves (including notions of monotonicity and concavity).
- analyze planar curves given in vector form including velocity and acceleration vectors.
- solve equations and situations for optimization, for both absolute (global) and relative (local) extrema.
- model problems involving rates of change, including related rates problems.
- use implicit differentiation for implicitly stated relations, functions and inverse functions.
- interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- explain the geometric interpretation of differential equations via slope fields and the relationship between slope fields and the derivatives of implicitly defined functions.
- determine the numerical solution of differential equations using Euler's Method.
- determine the derivatives of functions, including polynomial, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- determine derivatives using the rules for sums, products, and quotients.
- determine derivatives using the chain rule and implicit differentiation.
- determine derivatives of parametric, polar and vector functions.

- describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic and trigonometric.
- analyze essential relations in a problem to determine possible functions that could model the situation.
- solve problems involving exponential growth, including logistic growth models.
- solve problems involving direct and inverse variation.
- understand and use optimization strategies, including linear programming.
- apply the concepts of limits to sequences and asymptotic behavior of functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- recognize that the slope of the tangent line to a curve represents the rate of change.
- use logarithms and vectors to solve problems.
- perform operations with logarithms.
- recognize the relationships between a conditional statement and its converse.
- test the validity of logical arguments.
- visualize three-dimensional objects from different perspectives and analyze cross-sections, surface area and volume.
- use Cartesian and polar systems to represent, analyze and solve geometric and measurement problems.
- examine rotations of plane figures using sketches, coordinates, and function notation to solve related geometric problems.
- use successive approximation, upper and lower bounds, and limits to solve measurement problems.
- use properties of similarity and techniques of trigonometry to make indirect measurements of lengths and angles to solve a variety of problems.

Sample Assessment

Released open-ended and multiple choice questions from past AP exams

Pacing

12 weeks

Unit 3: Integrals

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

Extended

1.1 Students should understand and describe patterns and functional relationships.

1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.

1.3a Students will use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

Extended

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

2.2a Students will investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.

Extended

3.1 Students should use properties and characteristics of two- and three-dimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems.

3.1a Students will use methods of deductive and inductive reasoning to make, test, and validate geometric conjectures.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.

3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.

3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is calculus and what role does it play as a tool in science, business, and other areas of study?
- What is an integral (definite and indefinite), how can it be determined and/or evaluated and how can it be applied to problems in the real world?
- Given data from a function or relation how can its derivative and/or integral be found or approximated?
- What is the structure of the Advanced Placement exam? How can students maximize their efforts to be successful on the exam, in addition to having knowledge of the course content?

Core Topics

- Interpretations and properties of definite integrals
- Applications of integrals
- Fundamental Theorem of Calculus
- Techniques of antidifferentiation
- Applications of antidifferentiation

Unit Objectives

Students will be able to:

- operate with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- demonstrate an understanding of the definite integral both as a limit of Riemann sums and as the net accumulation of a rate of change and should be able to use integrals to solve a variety of problems.
- demonstrate an understanding of the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurements.

- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Skill Objectives

Students will:

- compute Riemann sums using left, right, and midpoint evaluation points.
- explain and write the definite integral as the limit of Riemann sums over equal subintervals.
- express the definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval $\int_a^b f'(x)dx = f(b) - f(a)$.
- use the basic properties of integrals, such as additivity and linearity.
- use Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.
- use integration to determine the area of a region.
- use integration to determine the volume of a solid with known cross section.
- use integration to determine the average value of a function.
- use integration to determine the distance traveled by a particle along a line.
- use integration to determine the length of a curve (including a curve given in parametric form).
  (The emphasis should always be on using the integral of a rate of change to give accumulated change or by setting up an approximating Riemann Sum and representing its limit as a definite integral.)
- use the Fundamental Theorem to evaluate definite integrals.
- use the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.
- determine antiderivatives of functions which follow directly from the derivatives of basic functions.
- determine antiderivatives by substitution of variables (including change of limits for definite integrals), integration by parts, and simple partial fractions (nonrepeating linear factors only).
- solve improper integrals (as limits of definite integrals).
- determine specific antiderivatives using initial conditions, including applications to motion along a line.
- explain L'Hospital's Rule and its use in determining convergence of improper integrals and series.
- solve separable differential equations and use them in modeling, including $y' = ky$ and exponential growth.
- solve logistic differential equations and use them in modeling population growth.

Sample Assessment

Released open-ended and multiple choice questions from past AP exams

Pacing

13 weeks

Unit 4: Polynomial Approximations and Series

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
Extended

1.1 Students should understand and describe patterns and functional relationships.
1.1a Students will model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.
1.2a Students will relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

1.3 Students should use operations, properties, and algebraic symbols to determine equivalence and solve problems.
1.3a Students will use and extend algebraic concepts to include real and complex numbers, vectors, and matrices.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
Extended

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.
2.2a Students will investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
Extended

3.1 Students should use properties and characteristics of two- and three-dimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems.
3.1a Students will use methods of deductive and inductive reasoning to make, test, and validate geometric conjectures.

3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.
3.2a Students will use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.
3.3a Students will approximate measurements that can not be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is the structure of the Advanced Placement exam? How can students maximize their efforts to be successful on the exam, in addition to having knowledge of the course content?
- Given data from a function or relation how can its derivative and/or integral be found or approximated?

Core Topics

- Series of Constants
- Maclaurin and Taylor Series

Unit Objectives

Students will be able to:

- operate with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- demonstrate an understanding of the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- model a written description of a physical situation with a function, a differential equation, or an integral.
- use technology to help solve problems, experiment, interpret results, and verify conclusions.
- determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurements.
- develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Skill Objectives

Students will:

- define a series as a sequence of partial sums and convergence as the limit of the sequence of partial sums.
- explain series through examples such as: decimal expansion, geometric series, the harmonic series, alternating series with error bound.
- relate the terms of series to the areas of rectangles and their relationship to improper integrals.

- use the integral test, where appropriate, to determine whether a series converges, including its use in testing the convergence of *p*-series.
- use the ratio test for convergence or divergence.
- use the limit comparison test for convergence or divergence.
- use the comparison test for convergence or divergence.
- use the alternating series test to determine if a series is convergent and if it is absolutely convergent.
- explain how Taylor and Maclaurin series approximate a polynomial with graphical demonstration.
- find Maclaurin series and the general Taylor Series centered at x = a.
- write Maclaurin series for the functions e^x, sin *x*, cos *x*, and 1/(1-x).
- manipulate Taylor series and use shortcuts to compute them, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
- write power series to represent appropriate functions.
- determine the radius and interval of convergence of power series.
- use the Lagrange error bound for Taylor Polynomials.
- describe and compare properties and classes of functions, including exponential, polynomial, rational, logarithmic and trigonometric.
- analyze essential relations in a problem to determine possible functions that could model the situation..
- solve problems involving exponential growth.
- solve problems involving direct and inverse variation.
- understand and use optimization strategies, including linear programming.
- apply the concepts of limits to sequences and asymptotic behavior of functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and line of symmetry for that function.
- recognize the effect of changes in parameters on the graphs of functions or relations.
- recognize that the slope of the tangent line to a curve represents the rate of change.
- combine, compose and invert functions.
- use logarithms to solve problems.
- perform operations with logarithms.
- recognize the relationships between a conditional statement and its converse
- use successive approximation, upper and lower bounds, and limits to solve measurement problems.

Sample Assessment

Released open-ended and multiple choice questions from past AP exams

Pacing

5 weeks