

Algebra 32 Summer Work Packet Review and Study Guide

This study guide is designed to accompany the Algebra 32 Summer Work Packet. Its purpose is to offer a review of the ten specific concepts covered in the Summer Work Packet. The mastery of these concepts prior to beginning Algebra 32 is essential. For each concept, there are explanations and examples as well as extra problems that students may choose to do on their own if they are experiencing difficulty and would like to reassure themselves that they have indeed mastered the concepts. Students are not required to print out this Guide and may use it simply as an online reference as they complete their summer work. However, printing this packet with the intention of including it in their Algebra 32 binder as a reference guide would serve them well throughout the entire course.

Topics Covered in this Packet:

- A. Order of Operations
- B. Fractions
- C. Simplifying Expressions
- D. Solving Equations
- E. Solving Inequalities
- F. Linear Graphs
- G. Linear Functions

A. Order of Operations

The rules for Order of Operations are as follows:

FIRST: Perform operations inside grouping symbols. Grouping symbols include parentheses (), brackets [], braces { }, radical symbols, absolute value symbols and fraction bars. If an expression contains more than one set of grouping symbols, simplify the expression inside the innermost set first. Follow the order of operations within that set of grouping symbols and then work outward.

SECOND: Simplify exponents.

THIRD: Perform multiplication and division from left to right. (Remember that a fraction bar also indicates division.)

FOURTH: Perform addition and subtraction from left to right.

Hint: You can use the well-known phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations. (Remember, however, that multiplication and division must be done in the order that they appear if they do not appear in parentheses. This is also true for addition and subtraction.)

Please	Parentheses
Excuse	Exponents
My	Multiplication
Dear	Division
Aunt	Addition
Sally	Subtraction

Example 1

Simplify.

$$-4^2 + 24 \div 3 \cdot 2$$

(Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not the "-".)

$$-16 + 24 \div 3 \cdot 2$$

$$-16 + 8 \cdot 2$$

$$-16 + 16$$

$$0$$

Example 2

Simplify.

$$4[25 - (5 - 2)^2]$$

$$4[25 - (3)^2]$$

$$4[25 - 9]$$

$$4 \cdot 16$$

$$64$$

Example 3

Simplify.

$$\frac{-22 - 2^2}{5 - 3}$$

$$\frac{(-22 - 2^2)}{(5 - 3)}$$

(Note that the fraction bar acts as a grouping symbol. You must simplify the numerator and denominator before dividing.)

$$\frac{-22 - 4}{2}$$

$$\frac{-26}{2}$$

$$-13$$

Try These:

Simplify.

1. $8 \div \frac{1}{2} \cdot 3$

2. $-20 \div [-2(4 + 1)]$

3. $\frac{5 + 2(-8)}{(-2)^3 - 3}$

4. $3\sqrt{50 - 1}$

5. $|4 - 7|^2 \div (-3)$

6. $\frac{2(-4) + 22}{4^2 - 9}$

B. Fractions

Adding & Subtracting Fractions

To add or subtract fractions, you must always have a common denominator. Once the denominators are the same, you add or subtract only the numerator to get your final answer. The common denominator of choice is the Lowest Common Denominator. If you make an effort to keep the numbers in fractions as small as possible, it will make subsequent calculations much easier.

Example 1

Perform the indicated operation.

$$\begin{aligned} & \frac{3}{4} - \frac{8}{16} \quad (\text{Note that in this case it is going to be much easier to have a common denominator of "4" rather than "32", "48" or "64".}) \\ &= \frac{3}{4} - \frac{2}{4} \\ &= \frac{1}{4} \end{aligned}$$

Example 2

Perform the indicated operation.

$$\begin{aligned} & \frac{5}{6} + \frac{3}{8} \\ &= \frac{20}{24} + \frac{9}{24} \\ &= \frac{29}{24} \end{aligned}$$

Try These:

Perform the indicated operation.

1. $\frac{8}{9} - \frac{4}{5}$

3. $\frac{7}{10} - \frac{3}{8}$

5. $\frac{1}{2} + \frac{3}{5} + \frac{1}{10}$

2. $\frac{3}{4} + \frac{1}{6}$

4. $\left(\frac{3}{4} - \frac{1}{3}\right) + \frac{7}{12}$

6. $\frac{1}{5} + \frac{1}{6} - \frac{7}{15}$

Multiplying Fractions

Unlike adding and subtracting, you do not need a common denominator to multiply fractions. To multiply fractions you multiply the numerators and then the denominators. Then, it is good practice to always reduce your final answer. When multiplying a fraction by a whole number, you can rewrite the whole number in fraction form by putting a "1" in the denominator.

Example 3

Perform the indicated operation.

$$\begin{aligned}\frac{3}{4} \times \frac{2}{5} \\ &= \frac{3 \times 2}{4 \times 5} \\ &= \frac{6}{20} \\ &= \frac{3}{10}\end{aligned}$$

Example 4

Perform the indicated operation.

$$\begin{aligned}4 \times \frac{3}{8} \\ &= \frac{4}{1} \times \frac{3}{8} \\ &= \frac{12}{8} \\ &= \frac{3}{2}\end{aligned}$$

Try These: Perform the indicated operation.

1. $\frac{1}{2} \times \frac{4}{7}$

3. $5\left(\frac{1}{3} + \frac{2}{5}\right)$

5. $\frac{4}{3} + \frac{2}{3} \times 4$

2. $\frac{7}{12} \times \frac{3}{4}$

4. $2\left(\frac{3}{4}\right)\left(\frac{2}{7}\right)$

6. $\frac{3}{4} \times 8$

Dividing Fractions

You have heard this a thousand times: "Dividing by a fraction is the same as multiplying by its reciprocal." To get the reciprocal of a fraction, you switch the numerator and the denominator. Another way of thinking of it is: a fraction multiplied by its reciprocal will always give you "1". For example the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$, and $\frac{3}{5} \times \frac{5}{3} = 1$. As always, to maintain good form, you must reduce your final answer to its simplest form.

Example 1

Perform the indicated operation.

$$\begin{aligned}\frac{3}{4} \div \frac{2}{3} \\ &= \frac{3}{4} \times \frac{3}{2} \\ &= \frac{9}{8}\end{aligned}$$

Example 2

Perform the indicated operation.

$$\begin{aligned}\frac{2}{3} \div 6 \\ &= \frac{2}{3} \times \frac{1}{6} \\ &= \frac{1}{9}\end{aligned}$$

Try These: Perform the indicated operation.

1. $\frac{11}{18} \div \frac{1}{3}$

3. $\frac{9}{10} \div \frac{6}{7}$

5. $\frac{4}{5} \times \frac{2}{3} \div \frac{8}{5}$

2. $\frac{2}{9} \div \frac{6}{7}$

4. $\left(\frac{3}{7} \div \frac{3}{5}\right) \div \frac{4}{7}$

6. $6\left(\frac{11}{18} \div \frac{1}{3}\right)$

C. Simplifying Expressions

To simplify algebraic expressions you need to apply the rules for the Order of Operations and collect like terms. Like terms are terms that contain the same variables raised to the same powers. Constants (numbers with no variable) are also like terms and can be simplified according to the order of operations. Typically, simplified expressions are written with the variables in descending order according to their exponents.

Distributive Property: $a(b + c) = ab + ac$ ex. $3(x - 4) = 3x - 12$

When simplifying algebraic expressions it is important to remember the Algebraic Properties of Equality. The Distributive Property is important and is also the property with which many students have difficulties. Remember that when the coefficient in front of parentheses is preceded by a negative, that negative must also be distributed through the parentheses. For example if we apply the distributive property to $x + 4(x - 3)$, the 4 is going to be distributed through the parentheses to get $x + 4x - 12$ or $5x - 12$. However if we apply the distributive property to $x - 4(x - 3)$, the -4 must be distributed to get $x - 4x - 12$ or $-3x - 12$.

Example 1

Simplify.

$$4x(1 - x) = (3x^2 - 2x + 7)$$

$$4x - 4x^2 - 3x^2 + 2x - 7$$

$$= 6x - 2x^2 - 7$$

$$= -2x^2 + 6x - 7$$

Example 2

Simplify.

$$(x - 2y) - 2(5x - y - 7)$$

$$= x - 2y - 10x + 2y + 14$$

$$= -9x + 14$$

Example 3

Simplify.

$$\frac{-12x^7 - 8x^5 + 24x^4 - 32x^2}{4x^2}$$

$$= -3x^5 - 2x^3 + 6x^2 - 8$$

Try These: Simplify.

1. $5(x+3)-7$

3.
$$\frac{\frac{2}{5}x^3 + \frac{1}{5}x^3 - \frac{1}{2}(3x^2 - 6x^2)}{10x}$$

5. $5 \cdot 4[3(x+9)+2]-7x$

2. $12x+8x+y-7x$

4. $24x+y^2+3x+2y^2$

6. $\frac{1}{2}[(10+x)-(-6+3x)]$

D. Solving Equations

To solve equations, we must isolate the variable in the equation to find the value of the variable that makes the equation true. We can think of an equation as a balanced scale. We must perform the same operation to both sides of the equation to keep it equal. When solving equations remember: "What you do to one side, you must do to the other." When solving linear equations, $0 = 0$ is an acceptable answer and is translated to mean that "x" can be any real number.

Example 1

Solve for x .

$$2(2x-1) = 3x+4$$

$$4x-2 = 3x+4$$

$$4x-3x = 4+2$$

$$x = 6$$

Example 2

Solve for x .

$$\frac{2x+3}{5} = \frac{x-1}{3} \quad \text{This is a proportion and can be solved by}$$

cross-multiplying.

$$3(2x+3) = 5(x-1)$$

$$6x+9 = 5x-5$$

$$6x-5x = -5-9$$

$$x = 14$$

Example 3

Solve for x .

$$4x^2 = 36$$

$$x^2 = 9 \quad \text{Remember that there are two answers here. Since "x" is squared, it's value can be both positive and negative.}$$

$$x = \pm 3$$

Try These:

Solve for x .

1. $6 - 7(x + 1) = -3(2 - x)$

2. $-5 - 7 - 3x = -x - 2(x + 6)$

3. $\frac{2}{3}x + \frac{1}{2} = \frac{3}{5}x - \frac{5}{6}$

4. $\frac{3x - 5}{4} = \frac{x - 5}{3}$

5. $4(3x + 1) - 7x = 6 + 5x - 2$

6. $4x + 2 [4 - 2(x + 2)] = 2x - 4$

E. Solving Inequalities

An inequality is a statement that two or more quantities are not equal. The statement gives information about how the value of one quantity is related to the other(s). Solving an inequality means finding the set of values for "x" that makes the statement true. Remember that when you divide both sides of an inequality by a negative, you must reverse the inequality sign.

ex. $3 < 5$, but $3(-2) > 5(-2)$

Thus it follows that in solving this inequality, you must reverse the sign when you isolate the "x":

$$3 - x > 5$$

$$-x > 5 - 3$$

$$-x > 2$$

Therefore, $x < -2$

Compound Inequalities are used to describe the union (OR) or intersection (AND) between three values. To solve compound inequalities, you must rewrite them as simple inequalities and then solve each "piece".

ex. $4 \leq x + 2 \leq 8$

This compound inequality can be written as two simple inequalities making them easier to solve.

Case 1:

$$x + 2 \geq 4$$

$$x \geq 2$$

Case 2:

$$x + 2 \leq 8$$

$$x \leq 6$$

According to these answers, x must be between 2 and 6, so the solution is:

$$2 \leq x \leq 6$$

Example 1

Solve for x.

$$4x \geq 7x + 6$$

$$4x - 7x \geq 6$$

$$-3x \geq 6$$
 Don't forget that when we divide by -3, we will have to reverse the sign.

$$x \leq -2$$

Example 2

Solve for x.

$$5 \leq 2x + 3 < 9$$
 This is a compound inequality, so we must consider two cases:

Case 1:

$$2x + 3 \geq 5$$

$$x \geq 1$$

Case 2:

$$2x + 3 < 9$$

$$x \leq 3$$

$$1 \leq x < 3$$
 Put these two solutions together.

Try These:

Solve for x.

1. $2(x-5) < -3x$

3. $-3(2x-4) \geq 2(5-3x)$

5. $5 < 3x-1 < 17$

2. $-2(x-7)-4-x < 8x+32$

4. $1 \leq 2x-5 \leq 7$

6. $5 \leq 4x-3 \leq 9$

F. Linear Graphs

Slope of a Line: The rate of change that compares the amount of change in a dependent variable (y) to the amount of change in an independent variable (x).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

A Few Things to Remember:

- Two lines that are parallel have the same slope.
- Two lines that are perpendicular have slopes that are negative reciprocals.
- The slope of a horizontal line is 0.
- The slope of a vertical line is undefined.
- Slope-Intercept form is $y = mx + b$.
- Standard form is $Ax + By = C$.

Example 1

Given the two points M(4, 2) and N(-2, -1) on a line, find the slope of the line \overline{MN} .

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - (-1)}{4 - (-2)}$$

$$= \frac{1}{2}$$

Intercepts:

The x -intercept is the point at which the graph crosses the x -axis. (In other words, it's the point at which $y = 0$.) The y -intercept is the point at which the graph crosses the y -axis. (In other words, it's the point at which $x = 0$.) Using intercepts is one of the easiest ways to graph a line on the coordinate plane.

ex. Let's say you want to graph the line defined by:

$$3x - 2y = 12$$

To find the x -intercept, set $y = 0$ and solve for x :

$$3x - 2(0) = 12$$

$$3x = 12$$

$$x = 4$$

Now, on the coordinate plane, go to 4 on the x -axis to graph the point at which this graph crosses the x -axis.

To find the y -intercept, set $x = 0$ and solve for y :

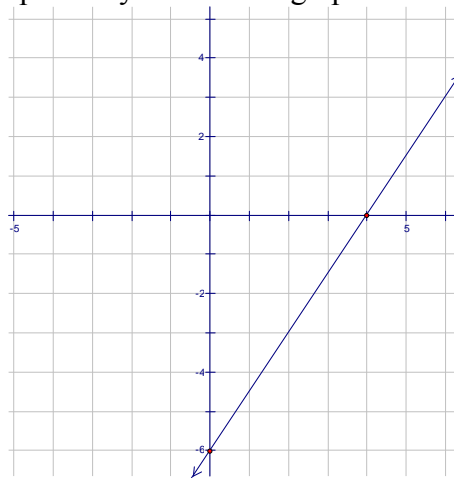
$$3(0) - 2y = 12$$

$$-2y = 12$$

$$y = -6$$

Now, on the coordinate plane, go to -6 on the y -axis to graph the point at which this graph crosses the y -axis.

Connect the x - and y -intercepts and you have the graph of the line $3x - 2y = 12$



Example 2

Find the intercepts for the line $\frac{1}{2}x = 4y + 4$

x-int.

$$\frac{1}{2}x = 4(0) + 4$$

$$\frac{1}{2}x = 4$$

$$x = 8$$

$$x\text{-int:}(8, 0)$$

y-int.

$$\frac{1}{2}(0) = 4y + 4$$

$$0 = 4y + 4$$

$$y = -1$$

$$y\text{-int:}(0, -1)$$

Slope-Intercept Form The Slope-Intercept form of the equation of a line is:

$$y = mx + b$$

Writing the equation of a line in this form gives you a lot of valuable information immediately because in this equation m is the slope of the line and b is the y-intercept.

Try These:

Find the slope of the line containing the two given points. Also, state the slope of a line that is perpendicular.

1. M(-1, 4), N(-1, -3)
2. M(0, 2), N(4, -1)
3. M(-6, 3), N(1, -4)

Find the slope as well as the x- and y-intercepts of the given lines.

4. $y = 4 - 2x$
5. $-2x - y = 4$
6. $x - 3y = -1$

G. Relations & Functions

A *relation* is a set of ordered pairs. The *domain* of a relation is the set of first coordinates (or x -values) of the ordered pairs. The *range* of a relation is the set of second coordinates (or y -values) of the ordered pairs.

ie. An example of a relation is $A = \{(1,5), (2,3), (3,2), (4,1)\}$

domain of $A = \{1, 2,3,4\}$

range of $A = \{5,3, 2,1\}$

A *function* is a special type of relation that pairs each domain value with exactly one range value.

Example 1:

Is the following relation a function?

$Y = \{(-4, 2), (-8, 2), (4,1), (5,1)\}$ Is every x -value paired with only one y -value?

Yes Y is a function.

Example 2: Is the following relation a function?

$X = \{(3,-2), (5,-1), (4,0), (3,1)\}$ Is every x -value only used once?

No, X is not a function because 3 is paired with two different y values (-2, & 1).