

## **Geometry 21 Summer Work Packet Review and Study Guide**

This study guide is designed to accompany the Geometry 21 Summer Work Packet. Its purpose is to offer a review of the ten specific concepts covered in the Summer Work Packet. The mastery of these concepts prior to beginning Geometry 21 is essential. For each concept, there are explanations and examples as well as extra problems that students may choose to do on their own if they are experiencing difficulty and would like to reassure themselves that they have indeed mastered the concepts. Students are not required to print out this Guide and may use it simply as an online reference as they complete their summer work. However, printing this packet with the intention of including it in their Geometry 21 binder as a reference guide would serve them well throughout the entire course.

Topics Covered in this Packet:

- A. Order of Operations
- B. Fractions
- C. Exponents
- D. Radicals
- E. Simplifying Expressions
- F. Solving Equations
- G. Solving Inequalities
- H. Linear Graphs
- I. Multiplying, Factoring and Solving Polynomial Expressions and Equations
- J. Solving Systems of Linear Equations

## A. Order of Operations

The rules for Order of Operations are as follows:

**FIRST:** Perform operations inside grouping symbols. Grouping symbols include parentheses ( ), brackets [ ], braces { }, radical symbols, absolute value symbols and fraction bars. If an expression contains more than one set of grouping symbols, simplify the expression inside the innermost set first. Follow the order of operations within that set of grouping symbols and then work outward.

**SECOND:** Simplify exponents.

**THIRD:** Perform multiplication and division from left to right. (Remember that a fraction bar also indicates division.)

**FOURTH:** Perform addition and subtraction from left to right.

Hint: You can use the well-known phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations. (Remember, however, that multiplication and division must be done in the order that they appear if they do not appear in parentheses. This is also true for addition and subtraction.)

Please	Parentheses
Excuse	Exponents
My	Multiplication
Dear	Division
Aunt	Addition
Sally	Subtraction

Example 1

Simplify.

$$-4^2 + 24 \div 3 \cdot 2$$

(Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not the "-".)

$$-16 + 24 \div 3 \cdot 2$$

$$-16 + 8 \cdot 2$$

$$-16 + 16$$

$$0$$

Example 2

Simplify.

$$4[25 - (5 - 2)^2]$$

$$4[25 - (3)^2]$$

$$4[25 - 9]$$

$$4 \cdot 16$$

$$64$$

Example 3

Simplify.

$$\frac{-22 - 2^2}{5 - 3}$$

$$\frac{(-22 - 2^2)}{(5 - 3)}$$

(Note that the fraction bar acts as a grouping symbol. You must simplify the numerator and denominator before dividing.)

$$\frac{-22 - 4}{2}$$

$$\frac{-26}{2}$$

$$-13$$

Try These:

Simplify.

1.  $8 \div \frac{1}{2} \cdot 3$

2.  $-20 \div [-2(4 + 1)]$

3.  $\frac{5 + 2(-8)}{(-2)^3 - 3}$

4.  $3\sqrt{50 - 1}$

5.  $|4 - 7|^2 \div (-3)$

6.  $\frac{2(-4) + 22}{4^2 - 9}$

Try These Answers:

1. 48

2. 2

3. 1

4. 21

5. -3

6. 2

## B. Fractions

### Adding & Subtracting Fractions

To add or subtract fractions, you must always have a common denominator. Once the denominators are the same, you add or subtract only the numerator to get your final answer. The common denominator of choice is the Lowest Common Denominator. If you make an effort to keep the numbers in fractions as small as possible, it will make subsequent calculations much easier.

#### Example 1

Perform the indicated operation.

$$\begin{aligned} \frac{3}{4} - \frac{8}{16} & \text{ (Note that in this case it is going to be much easier to have a common denominator of "4" rather than "32", "48" or "64".)} \\ &= \frac{3}{4} - \frac{2}{4} \\ &= \frac{1}{4} \end{aligned}$$

#### Example 2

Perform the indicated operation.

$$\begin{aligned} \frac{5}{6} + \frac{3}{8} \\ &= \frac{20}{24} + \frac{9}{24} \\ &= \frac{29}{24} \end{aligned}$$

Try These:

Perform the indicated operation.

1.  $\frac{8}{9} - \frac{4}{5}$

2.  $\frac{3}{4} + \frac{1}{6}$

3.  $\frac{7}{10} - \frac{3}{8}$

4.  $\left(\frac{3}{4} - \frac{1}{3}\right) + \frac{7}{12}$

5.  $\frac{1}{2} + \frac{3}{5} + \frac{1}{10}$

6.  $\frac{1}{5} + \frac{1}{6} - \frac{7}{15}$

Try These Answers:

1.  $\frac{4}{45}$

2.  $\frac{11}{12}$

3.  $\frac{13}{40}$

4. 1

5.  $\frac{6}{5}$

6.  $-\frac{1}{10}$

### Multiplying Fractions

Unlike adding and subtracting, you do not need a common denominator to multiply fractions. To multiply fractions you multiply the numerators and then the denominators. Then, it is good practice to always reduce your final answer. When multiplying a fraction by a whole number, you can rewrite the whole number in fraction form by putting a "1" in the denominator.

#### Example 3

Perform the indicated operation.

$$\begin{aligned}\frac{3}{4} \times \frac{2}{5} \\ &= \frac{3 \times 2}{4 \times 5} \\ &= \frac{6}{20} \\ &= \frac{3}{10}\end{aligned}$$

#### Example 4

Perform the indicated operation.

$$\begin{aligned}4 \times \frac{3}{8} \\ &= \frac{4}{1} \times \frac{3}{8} \\ &= \frac{12}{8} \\ &= \frac{3}{2}\end{aligned}$$

Try These: Perform the indicated operation.

1.  $\frac{1}{2} \times \frac{4}{7}$

2.  $\frac{7}{12} \times \frac{3}{4}$

3.  $5\left(\frac{1}{3} + \frac{2}{5}\right)$

4.  $2\left(\frac{3}{4}\right)\left(\frac{2}{7}\right)$

5.  $\frac{4}{3} + \frac{2}{3} \times 4$

6.  $\frac{3}{4} \times 8$

Try These Answers:

1.  $\frac{2}{7}$

2.  $\frac{7}{16}$

3.  $\frac{11}{3}$

4.  $\frac{3}{7}$

5. 4

6. 6

### Dividing Fractions

You have heard this a thousand times: "Dividing by a fraction is the same as multiplying by its reciprocal." To get the reciprocal of a fraction, you switch the numerator and the denominator. Another way of thinking of it is: a fraction multiplied by its reciprocal will always give you "1". For example the reciprocal of  $\frac{3}{5}$  is  $\frac{5}{3}$ , and  $\frac{3}{5} \times \frac{5}{3} = 1$ . As always, to maintain good form, you must reduce your final answer to its simplest form.

#### Example 1

Perform the indicated operation.

$$\begin{aligned}\frac{3}{4} \div \frac{2}{3} \\ &= \frac{3}{4} \times \frac{3}{2} \\ &= \frac{9}{8}\end{aligned}$$

#### Example 2

Perform the indicated operation.

$$\begin{aligned}\frac{2}{3} \div 6 \\ &= \frac{2}{3} \times \frac{1}{6} \\ &= \frac{1}{9}\end{aligned}$$

Try These: Perform the indicated operation.

1.  $\frac{11}{18} \div \frac{1}{3}$

3.  $\frac{9}{10} \div \frac{6}{7}$

5.  $\frac{4}{5} \times \frac{2}{3} \div \frac{8}{5}$

2.  $\frac{2}{9} \div \frac{6}{7}$

4.  $\left(\frac{3}{7} \div \frac{3}{5}\right) \div \frac{4}{7}$

6.  $6\left(\frac{11}{18} \div \frac{1}{3}\right)$

Try These Answers:

1.  $\frac{11}{6}$

2.  $\frac{7}{27}$

3.  $\frac{21}{20}$

4.  $\frac{5}{4}$

5.  $\frac{1}{3}$

6. 11

## C. Exponents

The properties of exponents are as follows:

Product of Powers	$x^m \cdot x^n = x^{m+n}$	$2^3 \cdot 2^4 = 2^7 = 128$
Power-of-a-Power:	$(x^m)^n = x^{mn}$	$(7^2)^3 = 7^6 = 117,649$
Power-of-a-Product	$(xy)^n = x^n y^n$	$(-3x)^2 = (-3)^2 x^2 = 9x^2$
Quotient-of-Powers:	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{3^8}{3^5} = 3^{8-5} = 3^3 = 27$
Positive Power of a Quotient:	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$
Negative Power of a Quotient:	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$	$\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$
Zero Power:	$a^0 = 1$	$\left(\frac{x^3 y^7}{4m^2 n}\right)^0 = 1$

### Example 1

Simplify each expression leaving no negative exponents.

$$\begin{aligned} & \frac{3x^5}{6x^{-2}} \\ &= \frac{(3x^5)(x^2)}{6} \\ &= \frac{3x^{10}}{6} \\ &= \frac{x^{10}}{2} \end{aligned}$$

### Example 2

Simplify each expression leaving no negative exponents.

$$\begin{aligned} & 3x(2x^2 y + 7y) \\ &= 6x^3 y + 21xy \end{aligned}$$

Example 3

Simplify each expression leaving no negative exponents.

$$\begin{aligned}(4x^2)^{-3} \\ &= \frac{1}{(4x^2)^3} \\ &= \frac{1}{4^3 x^6} \\ &= \frac{1}{64x^6}\end{aligned}$$

Example 4

Simplify each expression leaving no negative exponents.

$$\begin{aligned}\frac{4x^{-1}y^2}{5x^{-3}y^3} \cdot \frac{5y^4}{6y^{-3}} \\ &= \frac{4x^3y^2}{5xy} \cdot \frac{5y^4y^3}{6} \\ &= \frac{4x^2}{5y} \cdot \frac{5y^7}{6} \\ &= \frac{2x^2y^6}{3}\end{aligned}$$

Try These: Simplify each expression leaving no negative exponents.

$$\begin{array}{ll} 1. \left(\frac{s^3t}{st^4}\right)^2 & 2. \left[\left(\frac{3mn}{2}\right)^{-1}\right]^{-4} \\ 3. \left(\frac{3c}{-2}\right)^{-1} \left(\frac{d}{4}\right)^{-2} & 4. -\left(\frac{-t}{3v}\right)^{-4} \\ 5. \left(\frac{6}{7}\right)^{-2} \left(\frac{4s}{6t}\right)^{-4} & 6. \left(\frac{3a}{2b}\right)^{-4} \end{array}$$

Try These Answers:

$$1. \frac{s^4}{t^6} \quad 2. \frac{81m^4n^4}{16} \quad 3. \frac{-32}{3cd^2} \quad 4. -\frac{81v^4}{t^4} \quad 5. \frac{441t^4}{64s^4} \quad 6. \frac{16b^4}{81a^4}$$



## D. Radicals

Some properties of radicals to know are:

Product Property	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	$\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$
Quotient Property	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$

A few other things to remember about radicals:

#1 You should know what numbers have nice square roots:

4, 9, 16, 25, 36, 49, 64, ...

#2 Never leave a perfect square inside a radical.

Example:

$$\sqrt{32} = \sqrt{4} \cdot \sqrt{8} = 2\sqrt{8} \quad \text{The square root of 8 still has a factor of 4 in it. This answer is incomplete and needs to be finished.}$$

#3 Never leave a radical in the denominator of a fraction.

Example:

$$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5} \quad \text{To get rid of the radical in the denominator, multiply the numerator and denominator by the radical}$$

Example 1

Evaluate the expression. Leave answer in exact form.

$$\begin{aligned} & \sqrt{27} \\ &= \sqrt{9} \cdot \sqrt{3} \\ &= 3\sqrt{3} \end{aligned}$$

Example 2

Evaluate the expression. Leave answer in exact form.

$$\begin{aligned} & \sqrt{\frac{162}{64}} \\ &= \frac{\sqrt{162}}{\sqrt{64}} \\ &= \frac{\sqrt{81} \cdot \sqrt{2}}{8} \\ &= \frac{9\sqrt{2}}{8} \end{aligned}$$

Example 3

Evaluate the expression. Leave answer in exact form.

$$\begin{aligned} & \sqrt{45} \cdot \sqrt{18} \\ &= (\sqrt{9} \cdot \sqrt{5})(\sqrt{9} \cdot \sqrt{2}) \\ &= (3\sqrt{5})(3\sqrt{2}) \\ &= 9\sqrt{10} \end{aligned}$$

Example 4

Evaluate the expression. Leave answer in exact form.

$$\begin{aligned} & \sqrt{\frac{45}{8}} \\ & \sqrt{\frac{45}{8}} = \frac{\sqrt{45}}{\sqrt{8}} = \frac{\sqrt{9} \cdot \sqrt{5}}{\sqrt{4} \cdot \sqrt{2}} \\ &= \frac{3\sqrt{5}}{2\sqrt{2}} = \frac{3\sqrt{5}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{3\sqrt{10}}{2 \cdot 2} = \frac{3\sqrt{10}}{4} \end{aligned}$$

Try These: Simplify Evaluate the expression. Leave answer in exact form.

1.  $\sqrt{18}$

2.  $\sqrt{\frac{8}{9}}$

3.  $\sqrt{98} \cdot \sqrt{8}$

4.  $\left(\sqrt{\frac{72}{25}}\right)(2\sqrt{8})$

5.  $\sqrt{\frac{20}{8}}$

6.  $5 \cdot \sqrt{\frac{36}{5}}$

Try These Answers:

1.  $3\sqrt{2}$

2.  $\frac{2\sqrt{2}}{3}$

3. 28

4.  $\frac{48}{5}$

5.  $\frac{\sqrt{10}}{2}$

6.  $6\sqrt{5}$

## E. Simplifying Expressions

To simplify algebraic expressions you need to apply the rules for the Order of Operations and collect like terms. Like terms are terms that contain the same variables raised to the same powers. Constants (numbers with no variable) are also like terms and can be simplified according to the order of operations. Typically, simplified expressions are written with the variables in descending order according to their exponents.

Distributive Property:  $a(b + c) = ab + ac$  ex.  $3(x - 4) = 3x - 12$

When simplifying algebraic expressions it is important to remember the Algebraic Properties of Equality. The Distributive Property is important and is also the property with which many students have difficulties. Remember that when the coefficient in front of parentheses is preceded by a negative, that negative must also be distributed through the parentheses. For example if we apply the distributive property to  $x + 4(x - 3)$ , the 4 is going to be distributed through the parentheses to get  $x + 4x - 12$  or  $5x - 12$ . However if we apply the distributive property to  $x - 4(x - 3)$ , the -4 must be distributed to get  $x - 4x - 12$  or  $-3x - 12$ .

Example 1

Simplify.

$$\begin{aligned}4x(1 - x) &= (3x^2 - 2x + 7) \\4x - 4x^2 - 3x^2 + 2x - 7 &= 6x - 2x^2 - 7 \\&= -2x^2 + 6x - 7\end{aligned}$$

Example 2

Simplify.

$$\begin{aligned}(x - 2y) - 2(5x - y - 7) \\&= x - 2y - 10x + 2y + 14 \\&= -9x + 14\end{aligned}$$

Example 3

Simplify.

$$\begin{aligned}\frac{-12x^7 - 8x^5 + 24x^4 - 32x^2}{4x^2} \\&= -3x^5 - 2x^3 + 6x^2 - 8\end{aligned}$$

Try These: Simplify.

- $5(x + 3) - 7$
- $12x + 8x + y - 7x$
- $\frac{\frac{2}{5}x^3 + \frac{3}{5}x^3 - \frac{1}{2}(2x^2 - 6x^2)}{x^2}$
- $24x + y^2 + 3x + 2y^2$
- $4[3(x + 9) + 2] - 7x$
- $\frac{1}{2}[(10 + x) - (-6 + 3x)]$

Try These Answers:

- $5x + 8$
- $13x + y$
- $x + 2$
- $3y^2 + 27x$
- $5x + 116$
- $8 - x$

## F. Solving Equations

To solve equations, we must isolate the variable in the equation to find the value of the variable that makes the equation true. We can think of an equation as a balanced scale. We must perform the same operation to both sides of the equation to keep it equal. When solving equations remember: "What you do to one side, you must do to the other." When solving linear equations,  $0 = 0$  is an acceptable answer and is translated to mean that "x" can be any real number.

Example 1

Solve for  $x$ .

$$2(2x - 1) = 3x + 4$$

$$4x - 2 = 3x + 4$$

$$4x - 3x = 4 + 2$$

$$x = 6$$

Example 2

Solve for  $x$ .

$$\frac{2x+3}{5} = \frac{x-1}{3} \quad \text{This is a proportion and can be solved by cross-multiplying.}$$

$$3(2x + 3) = 5(x - 1)$$

$$6x + 9 = 5x - 5$$

$$6x - 5x = -5 - 9$$

$$x = 14$$

Example 3

Solve for  $x$ .

$$4x^2 = 36$$

$$x^2 = 9 \quad \text{Remember that there are two answers here. Since "x" is squared, it's value can be both positive and negative.}$$

$$x = \pm 3$$

Try These:

Solve for  $x$ .

1.  $6 - 7(x + 1) = -3(2 - x)$

2.  $-5 - 7 - 3x = -x - 2(x + 6)$

3.  $\frac{2}{3}x + \frac{1}{2} = \frac{3}{5}x - \frac{5}{6}$

4.  $\frac{3x - 5}{4} = \frac{x - 5}{3}$

5.  $4(3x + 1) - 7x = 6 + 5x - 2$

6.  $4x + 2 [4 - 2(x + 2)] = 2x - 4$

Try These Answers:

1.  $x = \frac{1}{2}$    2.  $x = \text{any real \#}$    3.  $x = -20$    4.  $x = -1$    5.  $x = \text{any real \#}$    6.  $x = 2$

## G. Solving Inequalities

An inequality is a statement that two or more quantities are not equal. The statement gives information about how the value of one quantity is related to the other(s). Solving an inequality means finding the set of values for "x" that makes the statement true. Remember that when you divide both sides of an inequality by a negative, you must reverse the inequality sign.

$$\text{ex. } 3 < 5, \text{ but } 3(-2) > 5(-2)$$

Thus it follows that in solving this inequality, you must reverse the sign when you isolate the "x":

$$3 - x > 5$$

$$-x > 5 - 3$$

$$-x > 2$$

Therefore,  $x < -2$

Compound Inequalities are used to describe the union (OR) or intersection(AND) between three values. To solve compound inequalities, you must rewrite them as simple inequalities and then solve each "piece".

$$\text{ex. } 4 \leq x + 2 \leq 8$$

This compound inequality can be written as two simple inequalities making them easier to solve.

Case 1:

$$x + 2 \geq 4$$

$$x \geq 2$$

Case 2:

$$x + 2 \leq 8$$

$$x \leq 6$$

According to these answers, x must be between 2 and 6, so the solution is:

$$2 \leq x \leq 6$$

Example 1

Solve for x.

$$4x \geq 7x + 6$$

$$4x - 7x \geq 6$$

$$-3x \geq 6 \quad \text{Don't forget that when we divide by -3, we will have to reverse the sign.}$$

$$x \leq -2$$

Example 2

Solve for x.

$$5 \leq 2x + 3 < 9 \quad \text{This is a compound inequality, so we must consider two cases:}$$

Case 1:

$$2x + 3 \geq 5$$

$$x \geq 1$$

Case 2:

$$2x + 3 < 9$$

$$x \leq 3$$

$$1 \leq x < 3 \quad \text{Put these two solutions together.}$$

Try These:

Solve for x.

$$1. \quad 2(x - 5) < -3x$$

$$2. \quad -2(x - 7) - 4 - x < 8x + 32$$

$$3. \quad -3(2x - 4) \geq 2(5 - 3x)$$

$$4. \quad 1 \leq 2x - 5 \leq 7$$

$$5. \quad 5 < 3x - 1 < 17$$

$$6. \quad 5 \leq 4x - 3 \leq 9$$

Try These Answers:

$$1. \quad x < 2 \quad 2. \quad x > -2 \quad 3. \quad x = \text{any real \#} \quad 4. \quad 3 \leq x \leq 6 \quad 5. \quad 2 < x < 6 \quad 6. \quad 2 \leq x \leq 3$$

## H. Linear Graphs

Slope of a Line: The rate of change that compares the amount of change in a dependent variable ( $y$ ) to the amount of change in an independent variable ( $x$ ).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

A Few Things to Remember:

- Two lines that are parallel have the same slope.
- Two lines that are perpendicular have slopes that are negative reciprocals.
- The slope of a horizontal line is 0.
- The slope of a vertical line is undefined.
- Slope-Intercept form is  $y = mx + b$ .
- Standard form is  $Ax + By = C$ .

Example 1

Given the two points  $M(4, 2)$  and  $N(-2, -1)$  on a line, find the slope of the line  $\overline{MN}$ .

$$\begin{aligned}\text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-1)}{4 - (-2)} \\ &= \frac{1}{2}\end{aligned}$$

Intercepts:

The  $x$ -intercept is the point at which the graph crosses the  $x$ -axis. (In other words, it's the point at which  $y = 0$ .) The  $y$ -intercept is the point at which the graph crosses the  $y$ -axis. (In other words, it's the point at which  $x = 0$ .) Using intercepts is one of the easiest ways to graph a line on the coordinate plane.

ex. Let's say you want to graph the line defined by:

$$3x - 2y = 12$$

To find the  $x$ -intercept, set  $y = 0$  and solve for  $x$ :

$$3x - 2(0) = 12$$

$$3x = 12$$

$$x = 4$$

Now, on the coordinate plane, go to 4 on the  $x$ -axis to graph the point at which this graph crosses the  $x$ -axis.

To find the  $y$ -intercept, set  $x = 0$  and solve for  $y$ :

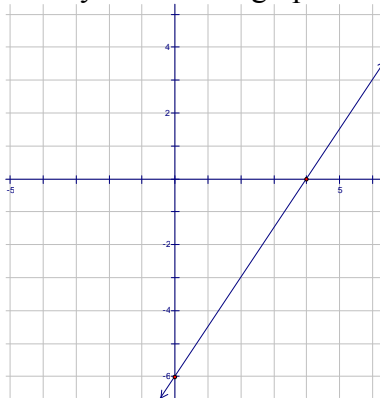
$$3(0) - 2y = 12$$

$$-2y = 12$$

$$y = -6$$

Now, on the coordinate plane, go to -6 on the y-axis to graph the point at which this graph crosses the y-axis.

Connect the x- and y-intercepts and you have the graph of the line  $3x - 2y = 12$



### Example 2

Find the intercepts for the line  $\frac{1}{2}x = 4y + 4$

x-int.

$$\frac{1}{2}x = 4(0) + 4$$

$$\frac{1}{2}x = 4$$

$$x = 8$$

$$x\text{-int:}(8, 0)$$

y-int.

$$\frac{1}{2}(0) = 4y + 4$$

$$0 = 4y + 4$$

$$y = -1$$

$$y\text{-int:}(0, -1)$$

Slope-Intercept Form The Slope-Intercept form of the equation of a line is:

$$y = mx + b$$

Writing the equation of a line in this form gives you a lot of valuable information immediately because in this equation  $m$  is the slope of the line and  $b$  is the y-intercept.

Try These:

Find the slope of the line containing the two given points. Also, state the slope of a line that is perpendicular.

1. M(-1, 4), N(-1, -3)
2. M(0, 2), N(4, -1)
3. M(-6, 3), N(1, -4)

Find the slope as well as the x- and y-intercepts of the given lines.

4.  $y = 4 - 2x$
5.  $-2x - y = 4$
6.  $x - 3y = -1$

Try These Answers:

1.  $m = \text{undefined}, m_{\perp} = 0$
2.  $m = -\frac{3}{4}, m_{\perp} = \frac{4}{3}$
3.  $m = -1, m_{\perp} = 1$

4.  $m = -2, x\text{-int} : 2, y\text{-int} : 4$
5.  $m = -2, x\text{-int} : -2, y\text{-int} : -4$

6.  $m = \frac{1}{3}, x\text{-int} : -1, y\text{-int} : \frac{1}{3}$

## I. Polynomial Expressions & Equations

A polynomial is defined as a monomial or a sum or difference of monomials.

### F.O.I.L. (First, Outer, Inner, Last)

FOIL is the method by which two binomials can be multiplied using the distributive property.

Example 1

Multiply the binomials using FOIL.

$$(x+3)(x+2) \quad \text{Take the first term of the first binomial and distribute it through the second binomial.}$$

*Then, take the second term in the first binomial and distribute it through the second binomial*

$$x^2 + 2x + 3x + 6$$

$$x^2 + 5x + 6$$

Example 2

Multiply the binomials using FOIL.

$$(x-8)(7x+4)$$

$$= 7x^2 + 4x - 56x - 32$$

$$= 7x^2 - 52x - 32$$

### Factoring:

A polynomial is in its factored form when it is written as a product of monomials and polynomials that cannot be factored further. There are several different types of factoring:

#### Greatest Common Factor:

Take out the largest factor that is common in each term and leave what is left of each term in parentheses.

Example 3

Factor.

$$8x^4 + 4x^3 - 2x^2 \quad 2x^2 \text{ is the largest factor that is common to each term.}$$

$$= 2x^2(4x^2 + 2x - 1) \quad \text{Use the distributive property to check this answer.}$$

#### Factoring Quadratic Trinomials ( $x^2+bx+c$ ):

When factoring quadratic trinomials with no coefficient in front of the  $x^2$ , ask yourself this question:

*What two numbers add to give "b" and multiply to give "c"?*

Example 4

Factor.

$$x^2 + 7x + 12$$

*What two numbers add to give "7" and multiply to give "12"?*

Obviously, the answer to this question is 3 and 4.

Therefore, the factors of  $x^2 + 7x + 12$  are  $(x + 3)(x + 4)$ .



Example 5

Factor.

$$x^2 - 10x + 16$$

*What two numbers add to give "-10" and multiply to give "16"?*

The answer to this question is -2 and -8.

Therefore, the factors of  $x^2 - 10x + 16$  are  $(x - 2)(x - 8)$ .

Example 6 Factor.

$$x^2 + 7x - 18$$

*What two numbers add to give "7" and multiply to give "-18"?*

The answer to this question is 9 and -2.

Therefore, the factors of  $x^2 + 7x - 18$  are  $(x + 9)(x - 2)$ .

Factoring Quadratic Trinomials ( $ax^2 + bx + c$ )

You may have learned this differently but if you don't remember, try these steps:

1-first, multiply  $ac$

2-find the factors of  $ac$  that add to get  $b$  (let's call these  $m$  and  $n$ )

3-expand the trinomial as follows  $ax^2 + mx + nx + c$

4-group the first two terms and the last two terms  $(ax^2 + mx) + (nx + c)$

5-factor out the greatest common factor in each group; the ( ) should both end up with the same expression inside

6-again, factor out the greatest common factor among the two remaining terms (the gcf should be the ( ))

Example 7

Factor  $2x^2 + x - 3$

*Multiply  $ac$  ( $2 \cdot -3$ ) = -6. Now, ask yourself which two numbers multiply to get -6 and add to get +1? Answer: 3 and -2. Now expand the quadratic as follows:*

$$2x^2 + 3x - 2x - 3$$

Now, group the first two terms and the last two terms and factor out the gcf.

$$(2x^2 + 3x) + (-2x - 3) \rightarrow x(2x + 3) - 1(2x + 3)$$

Notice that the ( ) now have the same expression inside. Again, factor out the gcf of the two remaining terms.

$$(2x + 3)(x - 1)$$

Example 8

Factor  $4x^2 - 4x - 35$

*Multiply  $ac$  ( $4 \cdot -35$ ) = -140. Now, ask yourself which two numbers multiply to get -140 and add to get -4? Answer: -14 and 10. Now expand the quadratic as follows:*

$$4x^2 - 14x + 10x - 35$$

Now, group the first two terms and the last two terms and factor out the gcf

$$(4x^2 - 14x) + (10x - 35) \rightarrow 2x(2x - 7) + 5(2x - 7)$$

Notice that the ( ) now have the same expression inside. Again, factor out the gcf of the two remaining terms.

$$(2x - 7)(2x + 5)$$

Example 8

Factor

Difference of Two Squares:

A polynomial is a difference of two squares if:

- There are two terms, one subtracted from the other.
- Both terms are perfect squares.

A polynomial that is a difference of two squares has the form:

$$a^2 - b^2 = (a - b)(a + b)$$

Example 8

Factor.

$$4x^2 - 25 \text{ In this case, } a = 2x \text{ and } b = 5.$$
$$= (2x - 5)(2x + 5)$$

Zero Product Property:

The Zero Product Property states that if the product of two quantities is zero, then at least one of the quantities equals zero. This property is particularly important when solving polynomial equations.

If  $ab = 0$ , then  $a = 0$  or  $b = 0$ .

The Zero Product Property can be applied when trying to solve equations such as:

$$x^2 + 7x + 10 = 0 \text{ Factoring this trinomial will give us the form } ab = 0.$$
$$(x + 2)(x + 5) = 0 \text{ According to the Zero Product Property, we can separate these factors.}$$
$$\begin{array}{l} x + 2 = 0 \\ x = -2 \end{array} \qquad \begin{array}{l} x + 5 = 0 \\ x = -5 \end{array}$$

Example 9

Solve for x.

$$x^2 = 16 \text{ Rewrite this so that the equation equals zero.}$$
$$x^2 - 16 = 0 \text{ This is a difference of squares that can be easily factored.}$$
$$(x + 4)(x - 4) = 0$$
$$x = -4 \text{ or } x = 4$$

Try These:

Factor each expression.

1.  $x^2 - 3x - 28$       2.  $16x^2 - 49y^2$       3.  $3x^3 - 3x^2 - 36x$       4.  $6x^2 + 5x - 4$

Solve each quadratic by factoring and using the Zero Product Property.

5.  $x^2 + 12x + 11 = 0$     6.  $2x^2 + 4x - 6 = 0$     7.  $4x^2 - 100 = 0$     8.  $2x^2 + 5x - 12 = 0$

Try These Answers:

1.  $(x - 7)(x + 4)$     2.  $(4x + 7y)(4x - 7y)$     3.  $3x(x - 4)(x + 3)$     4.  $(2x + 1)(3x - 4)$   
5.  $x = -11$  or  $-1$     6.  $x = -3$  or  $1$       7.  $x = 5$  or  $-5$       8.  $x = \frac{3}{2}, x = -4$

### The Quadratic Formula:

The Quadratic Formula is the only method that can be used to solve any quadratic equation. To find the value of "x" in a quadratic of the form  $ax^2 + bx + c = 0$ , use the following:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Example 10

Solve using the quadratic formula.

$$2x^2 = x - 3 \quad \text{We need to rewrite this to put it in the form } ax^2 + bx + c = 0.$$

$$x^2 - 2x - 3 = 0$$

$$a = 1$$

$$b = -2$$

$$c = -3$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - (-12)}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$x = \frac{2 \pm 4}{2}$$

$$x = \frac{2+4}{2} \quad \text{and} \quad x = \frac{2-4}{2}$$

$$x = 3 \quad \quad \quad x = -1$$

Try these:

Solve using the quadratic formula:

1.  $-2x^2 + 6x + 9 = 0$

2.  $4x^2 - 13x + 3 = 0$

3.  $x^2 + 7x - 2 = 0$

Try these answers:

1.  $x = \frac{3 \pm 3\sqrt{3}}{2}$

2.  $x = \frac{1}{4}, x = 3$

3.  $x = \frac{-7 \pm \sqrt{57}}{2}$

## J. Solving Systems of Equations

There are many ways to solve systems of equations (two or more equations with the same variables). We will focus on three of them.

### Solving By Graphing:

To solve a system of equations by graphing, simply graph each equation using the method you prefer and find the point(s) that they intersect. If the lines intersect in one point then there is one solution, the point of intersection. If the lines are parallel then there is no solution for the system. If the lines are the same line then there are infinitely many solutions (meaning that any point on the line is a solution).

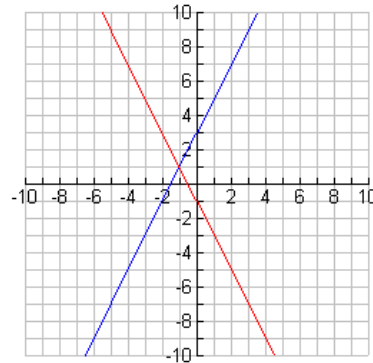
#### Example 1

Solve the system of equations by graphing.

$$y = 2x + 3$$

$$y = -2x - 1$$

In the graph at the right, the two lines intersect at the point  $(-1, 1)$ . So, that is the solution to the system.



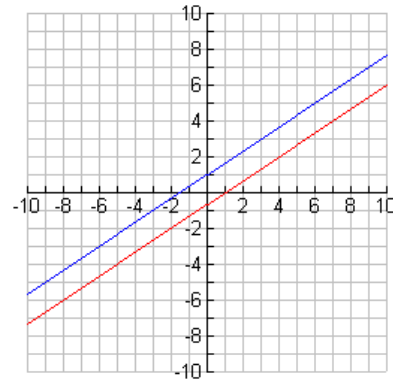
#### Example 2

Solve the system of equations by graphing.

$$y = \frac{2}{3}x + 1$$

$$3y = 2x - 2$$

In the graph at the right, the two lines are parallel. Thus, they never intersect. So, this system of equations has NO solution.



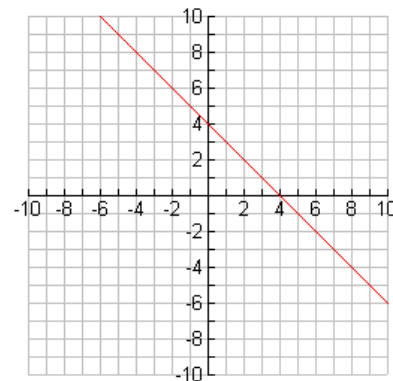
#### Example 3

Solve the system of equations by graphing.

$$y = -x + 4$$

$$3x + 3y = 12$$

In the graph at the right, the two equations are the same line. Thus, they intersect at every point on the line. So, this system of equations has infinitely many solutions.



### Solving by Substitution:

To solve a system by substitution, you need to solve one of the equations for one of the variables. Then, you substitute what you get for the variable into the OTHER equation. Then, solve for the remaining variable. Finally, substitute the value of the first variable into either equation and solve for the other variable.

#### Example 4

Solve the system of equations using substitution.

$$y = 2x + 3$$

$$y = -2x - 1$$

Here, both equations are already solved for y, so substitute  $(-2x-1)$  for y in the first equation.

$$-2x - 1 = 2x + 3$$

$$-4 = 4x$$

$$-1 = x$$

Now that we've solved for x, substitute  
What we got for x into either equation  
And solve for y.

$$y = 2x + 3$$

$$y = 2(-1) + 3$$

$$y = 1$$

So, the solution to this system of equations is  $(-1, 1)$

#### Example 5

Solve the system of equations using substitution.

$$2y = 5x + 12$$

$$4y - 10x = 8$$

First, solve the first equation for y  $y = \frac{5}{2}x + 6$  and then substitute what you get into y in the other equation.

$$4\left(\frac{5}{2}x + 6\right) - 10x = 8$$

Here, when we simplify, we get  $24=8$ , which is never true. Thus, there is no solution to this system.

$$10x + 24 - 10x = 8$$

$$24 = 8$$

There is NO solution to this system.

#### Example 6

Solve the system of equations using substitution.

$$2y + 4x = 8$$

$$y = -2x + 4$$

Here, the second equation is already solved for y, so simply substitute  $-2x+4$  in for y in the first equation and solve.

$$2(-2x + 4) + 4x = 8$$

$$-4x + 8 + 4x = 8$$

Here, we get  $8=8$  which is always true. Thus, there are infinitely many solutions to this system.

$$8 = 8$$

So, there are infinitely many solutions to this system.

### Solving by Linear Combinations:

To solve a system of equations using linear combinations, first you need to get the variables, constant, and equal sign lined up into columns. Next, you need to get one of the variables to have the same coefficient (number in front of the variable) with opposite signs in each of the equations. Sometimes, this will already be done for you. Sometimes, you will need to multiply one or both of the equations to get the same coefficient, opposite signs. Then, you “add” the two equations together. This will eliminate one of the variables. Next, you solve for the remaining variable. Finally, take the value of the first variable and substitute it into either equation and solve for the other variable.

#### Example 7

Solve the system of equations using linear combinations.

$$y - 2x = 3$$

$$y + 2x = -1$$

Here, the equations are already lined up for you and  $x$  already has the same coefficient with opposite signs. So, just add the two equations together and solve for  $y$ .

$$\begin{array}{r} y - 2x = 3 \\ + y + 2x = -1 \\ \hline 2y = 2 \\ y = 1 \end{array}$$

When you add the two equations together, the  $x$ 's cancel out. So, solve for  $y$  which is the remaining variable. Now, take what you got for  $y$  and substitute it into either equation and solve for  $x$ .

$$\begin{array}{r} (1) + 2x = -1 \\ 2x = -2 \\ x = -1 \end{array}$$

So, the solution to this system is  $(-1, 1)$ .

#### Example 8

Solve the system of equations using linear combinations.

$$9x - 4y = -18$$

$$-3x + 8y = 6$$

Neither variable has the same coefficient with opposite signs, so you need to multiply one or both of the equations to get the same coefficient opposite sign. Here, you could either multiply the first equation by 2 or the second equation by 3. I will multiply the first equation by 2...

$$\begin{array}{r} 18x - 8y = -36 \\ + -3x + 8y = 6 \\ \hline 15x = -30 \\ x = -2 \end{array}$$

Now, substitute -2 in for  $x$  in either equation and solve for  $y$ .

$$\begin{array}{r} -3(-2) + 8y = 6 \\ 6 + 8y = 6 \\ 8y = 0 \\ y = 0 \end{array}$$

So, the solution to this system is  $(-2, 0)$

Try These:

Solve each of the following systems using any method you prefer.

1. 
$$\begin{array}{l} y = 2x + 3 \\ y = 5x + 9 \end{array}$$

2. 
$$\begin{array}{l} y = 5 - x \\ x - y = 7 \end{array}$$

3. 
$$\begin{array}{l} x - 2y = 8 \\ -x + 3y = -15 \end{array}$$

4. 
$$\begin{array}{l} 4x + 5y = -2 \\ 5x - 4y = -23 \end{array}$$

Try These Answers:

1.  $(-6, -7)$

2.  $(6, -1)$

3.  $(-2, -1)$

4.  $(-3, 2)$