Geometry 22 Summer Work Packet
Review and Study Guide

This study guide is designed to accompany the Geometry 22 Summer Work Packet. Its purpose is to offer a review of the specific concepts covered in the Summer Work Packet. The mastery of these concepts prior to beginning Geometry 22 is essential. For each concept, there are explanations and examples as well as extra problems that students may choose to do on their own if they are experiencing difficulty and or wish to test themselves.

Topics Covered in this Packet:

A. Order of Operations

The rules for Order of Operations are as follows:

FIRST: Perform operations for groups made by parentheses ( ), brackets [ ] , braces { } , radical symbols , absolute value symbols and fraction bars. Work from the inside out: If an expression has groups within groups, simplify the expression for the innermost group first. Follow the order of operations within that group and then work outward.

SECOND: Simplify exponents.

THIRD: Perform multiplication and division from left to right. (So, if a multiplication and division are next to each other, do them left to right). A fraction is a division.

FOURTH: Perform addition and subtraction from left to right.

Hint: Use the phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations.

Please	Parentheses
Excuse	Exponents
My	Multiplication
Dear	Division
Aunt	Addition
Sally	Subtraction

Example 1

Simplify.

$-4^2 + 24 \div 3 \cdot 2$ (Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not the "-".)

$-16 + 24 \div 3 \cdot 2$

$-16 + 8 \cdot 2$

$-16 + 16$

0

Example 2

Simplify.

$4\left[25 - (5-2)^2\right]$

$4\left[25 - (3)^2\right]$

$4[25 - 9]$

$4 \cdot 16$

64

Example 3
Simplify.

$$\frac{-22-2^2}{5-3}$$

$$\frac{\left(-22-2^2\right)}{\left(5-3\right)}$$ (Note that the fraction bar acts as a grouping symbol. You must simplify the numerator and denominator before dividing.)

$$\frac{-22-4}{2}$$

$$\frac{-26}{2}$$

$$-13$$

Try These:
Simplify.

1. $8 \div 2 - 3$
2. $24 \div (2 \times (4 - 1))$
3. $\frac{11\times2+2\times8}{(-4)^2+3}$
4. $3\sqrt{50-1}$
5. $8 \times 4 - (6 \div 3 + (8 - 6)^2)$
6. $\frac{2(-4)+22}{4^2-9}$

Try These Answers:
1. 1 2. 4 3. 2 4. 21 5. 26 6. 2

B. Fractions, Decimals, and Percents

Adding & Subtracting Fractions
To add or subtract fractions, you must always have a common denominator. Once the denominators are the same, you add or subtract only the numerator to get your final answer. Fractions should always be written in most reduced from. Also, fractions should not be left as improper fractions. They should be written as mixed numbers when needed.

Example 1
Perform the indicated operation.

$$=\frac{3}{4}-\frac{7}{16}$$

$$=\frac{3}{4}*\frac{4}{4}-\frac{7}{16}$$

$$=\frac{12}{16}-\frac{7}{16}$$

$$=\frac{5}{16}$$

Example 2
Perform the indicated operation.

$$= \frac{2}{3} + \frac{4}{5}$$

$$= \frac{2}{3} * \frac{5}{5} + \frac{4}{5} * \frac{3}{3}$$

$$= \frac{10}{15} + \frac{12}{15}$$

$$= \frac{27}{15} = 1\frac{12}{15} = 1\frac{4}{5}$$

Try These: Perform the indicated operation.

1. $\frac{8}{9} - \frac{4}{5}$
2. $\frac{3}{4} + \frac{1}{6}$
3. $\frac{7}{10} - \frac{3}{8}$
4. $\frac{1}{2} + \frac{3}{5} + \frac{1}{10}$

Try These Answers:

1. $\frac{4}{45}$ 2. $\frac{11}{12}$ 3. $\frac{13}{40}$ 4. $1\frac{1}{5}$

Multiplying Fractions

You do not need a common denominator to multiply fractions. To multiply fractions you multiply across, multiplying the numerators and then multiplying the denominators. Always reduce your final answer. When multiplying a fraction by a whole number, you can rewrite the whole number in fraction form by giving it a denominator of "1". To multiply mixed numbers rewrite the mixed number as an improper fractions.

Example 3

Perform the indicated operation.

$$\frac{3}{4} \times \frac{2}{5}$$

$$= \frac{6}{20} = \frac{3}{10}$$

Example 4

Perform the indicated operation.

$$4 \times \frac{3}{8}$$

$$\frac{4}{1} \times \frac{3}{8}$$

$$= \frac{12}{8} = \frac{3}{2} = 1\frac{1}{2}$$

Example 5

Perform the indicated operation.

$$2\frac{2}{5} \times \frac{3}{4}$$

$$= \frac{12}{5} \times \frac{3}{4}$$

$$= \frac{36}{20} = 1\frac{16}{20} = 1\frac{4}{5}$$

Try These: Perform the indicated operation.

1. $\frac{1}{2} \times \frac{4}{7}$
2. $\frac{7}{12} \times \frac{3}{4}$
3. $2 \times \frac{7}{9}$
4. $5\left(\frac{1}{3} + \frac{2}{5}\right)$
5. $2\frac{3}{4} \times \frac{4}{9}$

Try These Answers:

1. $\frac{2}{7}$ 2. $\frac{7}{16}$ 3. $1\frac{5}{9}$ 4. $3\frac{2}{3}$ 5. $1\frac{2}{9}$

Dividing Fractions

Remember the rule "Invert and multiply." $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \times \frac{D}{C}$

Example 1

Perform the indicated operation.

$$\frac{3}{4} \div \frac{2}{3}$$

$$= \frac{3}{4} \times \frac{3}{2}$$

$$= \frac{9}{8} = 1\frac{1}{8}$$

Example 2

Perform the indicated operation.

$$\frac{2}{3} \div 6$$

$$= \frac{2}{3} \times \frac{1}{6}$$

$$= \frac{2}{18} = \frac{1}{9}$$

Try These: Perform the indicated operation.

1. $\frac{11}{18} \div \frac{1}{3}$
2. $\frac{2}{9} \div \frac{6}{7}$
3. $\frac{9}{10} \div \frac{6}{7}$
4. $\frac{4}{5} \times \frac{2}{3} \div \frac{8}{5}$

Try These Answers:

1. $1\frac{5}{6}$ 2. $\frac{7}{27}$ 3. $1\frac{1}{20}$ 4. $\frac{1}{3}$

Changing amongst fractions, decimals, and percents

There are three formats that you can use for writing any number. You need to be able to change amongst them. Percents are an important format because people describe a lot of different mathematical relationships using percents.

Converting fractions to decimals: Divide
Converting decimals to percents: Move the decimal point two places to the right
Converting percents to decimals: Move the decimal point two places to the left
To calculate a certain percent of a number:

- Convert the percent to a decimal
- Multiply the number by the decimal

To calculate what number X is a certain percent of:

- Convert the percent to a decimal
- Divide the number by the percent

Example 1

What is ¾ as a decimal? 3÷4 = .75

Example 2

What is .667 as a percent? 66.7%

Example 3

What is 44.3% as a decimal? .443.

Example 4

What is 40% of 80?

.4 * 80 = 32

Example 5

60 is 30% of what number?

$$\frac{60}{.30} = 200$$

Try these:

1. What is 2/5 as a decimal?
2. What is .345 as a percent?
3. What is 86.2% as decimal?
4. What is 25% of 40?
5. 22 is 11% of what number?

Try These Answers:

1. .4
2. 34.5%
3. .862
4. 10
5. 200

C. Radicals

Radicals can be broken up put back together by the following rules:

Product Property	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	$\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$ OR $\sqrt{2} \cdot \sqrt{8} = \sqrt{16} = \sqrt{4}$
Quotient Property	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$ OR $\frac{\sqrt{180}}{\sqrt{5}} = \sqrt{\frac{180}{5}} = \sqrt{36} = 6$

A few other things to remember about radicals:

#1 Know as many perfect squares as you can: 4, 9, 16, 25, 36, 49, 64, …

#2 Reduce radicals to simplest radical form by removing all the perfect squares that you recognize as factors of the number. The radical asks, "What is the square root of what is inside me?" We know the answer for any factors of the number that are perfect squares, so we can answer the radicals question for those parts and take them out.

Example:

$$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$$

#3 Eliminate radicals in the denominator of a fraction by rationalizing the denominator. (Never leave a radical in your basement; you never know what they will do down there.)

Example: $\sqrt{\frac{9}{5}} = \frac{\sqrt{9}}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$

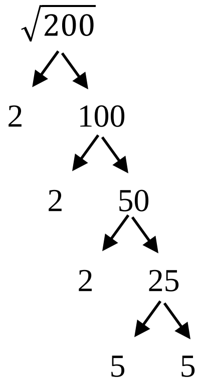
To get rid of a radical in the denominator of a fraction, multiply both the numerator and denominator of the fraction by the radical that appears in the denominator.

#4 Radicals can be eliminated by squaring. (As they found in the sixties, one square and one radical will eliminate each other.)

$$\left(\sqrt{7}\right)^2 = 7 \quad \{\sqrt{7} \times \sqrt{7} = \sqrt{49} = 7\}$$

$$\sqrt{7^2} = 7$$

<u>Reducing radicals</u> Radicals can be reduced either by simply recognizing perfect squares within the radical (see #2 above), or by finding perfect squares by factoring:



So, $\sqrt{200} =$

$$\sqrt{2 \times 2 \times 2 \times 5 \times 5} =$$
$$\sqrt{2} \times \sqrt{2 \times 2} \times \sqrt{5 \times 5} =$$
$$\sqrt{2} \times 2 \times 5 =$$
$$10\sqrt{2}$$

Any number times itself makes a perfect square that can be taken out of the radical.

Example 1
Re-write the expression in simplest radical form.

$\sqrt{45}$

$\sqrt{3 \times 3 \times 5}$

$\sqrt{3 \times 3} \times \sqrt{5}$

$3\sqrt{5}$

Example 2
Re-write the expression in simplest radical form.

$\sqrt{27}$

$= \sqrt{9} \cdot \sqrt{3}$

$= 3\sqrt{3}$

Example 3
Re-write the expression in simplest radical form.

$$\sqrt{\frac{162}{5}} = \frac{\sqrt{162}}{\sqrt{5}} = \frac{\sqrt{81}\sqrt{2}}{\sqrt{5}} = \frac{9\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{9\sqrt{10}}{5}$$

Example 4
Evaluate the expression. Leave answer in exact form.

$$\sqrt{12 \times 15^2} = \sqrt{12}\sqrt{15^2} = 15\sqrt{12} = 15\sqrt{2 \times 2 \times 3} = 30\sqrt{3}$$

Try These: Simplify Evaluate the expression. Leave answer in exact form.

1. $\sqrt{18}$

2. $\sqrt{\frac{8}{9}}$

3. $\sqrt{98} \cdot \sqrt{8}$

4. $\sqrt{\frac{20}{8}}$

5. $5\sqrt{\frac{36}{5}}$

6. $\sqrt{16^2 \times 8^2 \times 9}$

Try These Answers:

1. $3\sqrt{2}$ 2. $\frac{2\sqrt{2}}{3}$ 3. 28 4. $\frac{\sqrt{10}}{2}$ 5. $6\sqrt{5}$ 6. 384

D. Simplifying Expressions

Simplifying algebraic expressions is the first step before solving an equation (as discussed in part E). To simplify:

- apply the rules for the order of operations
- combine like terms
- distribute correctly

Like terms are terms that contain the same variables raised to the same powers.

$2x + 5x = 7x$

$x^2 + -4x^2 = 5x^2$

$4x^3 + 2x$ cannot be simplified

$x + y$ cannot be simplified

Constants (numbers with no variable) are also like terms and can be combined according to the order of operations.

Distributive Property: $a(b + c) = ab + ac$ ex. $3(x - 4) = 3x - 12$

Remember that when the coefficient in front of parentheses is negative, that negative must also be distributed through the parentheses.

ex. $-2(4+x) = -8 - 2x$

ex. $x - 4(x - 3) = x - 4x + 12 = -3x + 12$

Example 1

Simplify both sides.

$3(2 - x) = x(x + 7)$ Distribute

$6 - 3x = x^2 + 7x$

Proceed to solving

Example 2

Simplify both sides

$x + 2x - 3x^2 = 5x - 3(x - 7)$ Distribute

$x + 2x - 3x^2 = 5x - 3x + 21$ Combine x terms

$3x - 3x^2 = 2x + 21$

Proceed to solving

Example 3
Simplify both sides.
$x(x + 2) – (4 + x) = 10 – 3(15 – 4) + 3x$ Distribute

$x^2 + 2x – 4 – 4x = 10 – 3(11) + 3x$ Combine x terms

$x^2 – 2x – 4 = 10 – 33 + 3x$ Combine constants

$x^2 – 2x – 4 = -23 + 3x$

Proceed to solving

Example 4
Simplify both sides.
$2y + 3x – 2y^2 + y = 4 + 3x – 4y + 9x – x^2$ Combine y terms and combine x terms

$3y + 3x – 2y^2 = 4 + 12x -4y - x^2$

Try These:
Simplify.
1. $5(x + 3) – 7 = 2(x + 2) – 4$
2. $24x + y^2 + 3x + 2y^2 = 4x – 2 + 2(2x – 5)$
3. $12x + 8x + y – 7x = 8 – 4(x + 2)$
4. $x(x +4) + 3x^2 = 8x^2 + 4x – x(x + 2)$

Try These Answers:
1. $5x + 8 = 2x$ 2. $27x + 3y^2 = 8x -12$ 3. $13x +y = -4x$ 4. $4x^2 + 4x = 7x^2 + 2x$

E. Solving Equations

To solve equations, we must use valid steps to re-write the equation as many times as needed until the equation shows what the value of the unknown is. We stop working the equation when the unknown is alone or isolated, so that one side of the equation reads something like "X =". The first step in working with an equation is to simplify each side by combining like terms or distributing when needed. After that we should think of using the equal sign as the balance point of a scale. To re-write the equation, we perform the same operation to both sides of the equation to keep it equal. When solving equations remember: "What you do to one side, you must do to the other."

Example 1
Solve for *x*.

$3x + 2 = 20$
-2 = -2
$3x = 18$
--- ----
3 3
$x = 6$

Example 2

Solve for *x*.

$4x + 2x - 7 = 23$

$6x - 7 = 23$

+7 +7

$6x = 30$

--- ---

6 6

$x = 5$

Example 3

Solve for *x*.

2(2x – 1) = 3x + 4

4x – 2 = 3x + 4

-3x -3x

x – 2 = 4

+2 +2

x = 6

Example 4

Solve for *x*.

$$\frac{x}{12} = \frac{2}{3}$$

This is a proportion and can be solved by cross-multiplying.

$$\frac{x}{12} \quad \frac{2}{3}$$

$3x = 24$

--- ---

3 3

$x = 8$

Example 5

Solve for *x*.

$$\frac{2x+3}{5} = \frac{x-1}{3}$$

This is a proportion and can be solved by cross-multiplying.

$$\frac{2x+3}{5} \quad \frac{x-1}{3}$$

$3(2x + 3) = 5(x - 1)$

$6x + 9 = 5x - 5$

-9 -9

$6x = 5x - 14$

-5x -5x

$x = -14$

Example 6

Solve for *x*.

$$\frac{5x+3}{3} = 16$$

$$\frac{3}{1} \times \frac{5x+3}{3} = \frac{16}{1} \times \frac{3}{1}$$

$5x + 3 = 48$

-3 -3

$5x = 45$

--- ---

5 5

$x = 9$

Try These:

Solve for *x*.

1. $4x + 6 = 46$
2. $12x - 10 - 4x = 230$
3. $8(4 - x) = 8$
4. $\frac{2x}{3} = \frac{24}{4}$
5. $\frac{8}{10x-2} = \frac{16}{14x+8}$
6. $\frac{14x+20}{13} = 8$

Try These Answers:

1. x=10 2. x = 30 3. x = 3 4. x = 9 5. x = 2 6. x = 6

F. Linear Graphs

Slope of a Line: The rate of change that compares the amount of change between two points on a line in the dependent variable (*y*) to the amount of change in the independent variable (*x*).

$$slope = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

A Few Things to Remember:

- Two lines that are parallel have the same slope.
- Two lines that are perpendicular have slopes that are negative reciprocals.
- The slope of a horizontal line is 0.
- The slope of a vertical line is undefined.
- Slope-Intercept form is y = mx + b .
- A line increasing (going up) to the right has a POSITIVE slope.
- A line decreasing (going down) to the right has a NEGATIVE slope.

Example 1 (slope formula)
Find the slope of the line that goes through the points (4, 2) and (-2, -1).

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

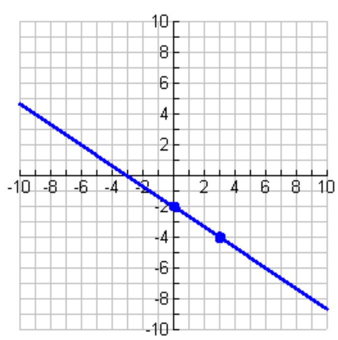
$$= \frac{2-(-1)}{4-(-2)}$$

$$= \frac{1}{2}$$

So, the line containing the points (4,2) and (-2,1) has a slope of ½ .

Example 2 (counting slope on graph)

If we are given the graph of a line, we can also ***count*** the $\frac{rise}{run}$ to find the slope.



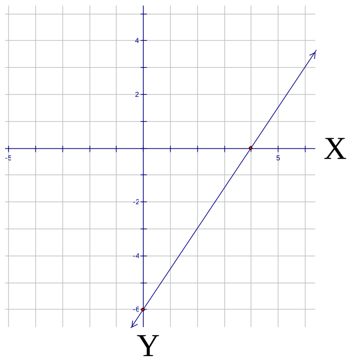
The 2 points that are marked on the graph are (0, -2) and (3, -4)

To get from one point to the other, we count down 2 and to the right 3.

$$\frac{rise}{run} = \frac{down 2}{right 3} = \frac{-2}{3} \quad \text{so the slope is } \frac{-2}{3}$$

Intercepts
The *x*-intercept is the point at which the graph crosses the *x*-axis. (y=0)
The y-intercept is the point at which the graph crosses the *y*-axis. (x=0)

Example 3 (finding intercepts on a graph)



x – intercept at point (4, 0)
y – intercept at point (0, - 6)

*To count the slope by looking at this graph, we could start at the x-intercept and count rise over run to the y-intercept.

Rise (up) 6,

Run (right) 4 → slope = $\frac{6}{4}$ which simplifies to $\frac{3}{2}$

Slope-Intercept Form

The Slope-Intercept form of the equation of a line is:

$$y = mx + b$$

where *m* is the slope of the line and *b* is the *y*-intercept.

Example 4

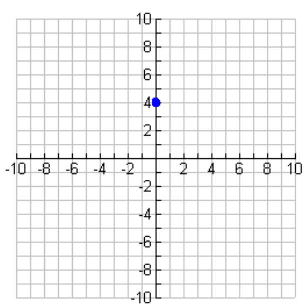
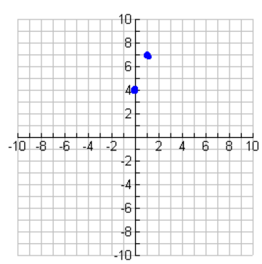
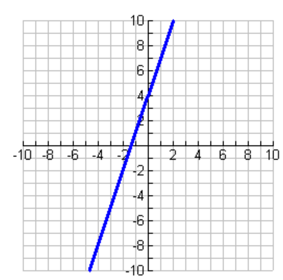
Given the equation y = 3x + 4, find the slope, y-intercept and graph the line

$y = mx + b$

$y = 3x + 4$ m=3, b=4

So we know that slope(m) = 3 and y-intercept(0,b) = (0,4)

Graph: $y = 3x + 4$

1)Plot the point at the y-intercept first b = 4 	2)Count slope from that point m = 3, so $\frac{rise}{run} = \frac{3}{1}$ up 3, right 1 , put another point 	3) connect your 2 points to draw the line 

Example 5
Write the equation in slope intercept form, then state the slope and y-intercept;

$2x - 4y = 8$ — we need to isolate y
$-2x \quad\quad -2x$ — by subtracting 2x from both sides first

$$\frac{-4y}{-4} = \frac{-2x}{-4} + \frac{8}{-4}$$ divide everything by -4

$$y = \frac{1}{2}x - 2$$ this is slope intercept form $y = mx + b$

so slope (m) = ½ and y-intercept(0,b) = (0,- 2)

Try These:

Find the slope of the line containing the two given points.

1. (-7, 4) and (-1, 4) 2. (0, 2) and (4, -1) 3. (-6, 3) and (1, -4)

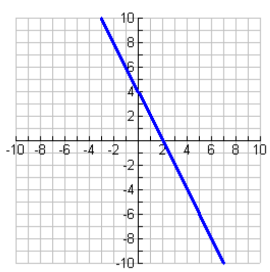
Find slope and y-intercept and graph the lines;

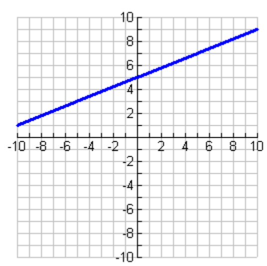
4. $y = -2x + 4$ 5. $y = 5 + \frac{2}{5}x$ 6. $x - 3y = -6$ (isolate y first)

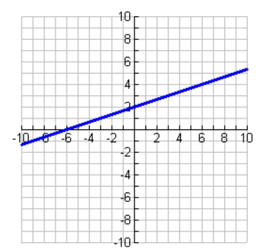
Try These Answers:

1. $m = 0$ 2. $m = -3/4$ 3. $m = -1$

4. $m = -2, b = 4$ 5. $m = \frac{2}{5}, b = 5$ 6. $m = \frac{1}{3}, b = 2$







G. Polynomial Expressions and Equations

Basic Operations

Examples

1. $y \bullet y = y^2$
2. $2x \bullet 3x = 6x^2$
3. $\frac{8x^2}{2x} = 4x$ (simplify numbers $\frac{8}{2}$ = 4, and variables $\frac{x^2}{x} = x$)

Distributive Property

1. $3x(4x – 5)$ Distribute 3x to everything in parentheses.
   $= 12x^2 - 15x$
2. $-7(2x^2 - 4x + 8)$ Distribute the – 7 to everything…watch your signs!
   $= -14x^2 + 28x - 56$

F.O.I.L. (First, Outer, Inner, Last)

FOIL is the method by which two binomials can be multiplied using the distributive property.

<u>Example 1</u>

Multiply the binomials using FOIL.

$(x+3)(x+2)$ Take the first term of the first binomial and distribute it through the second binomial.

$= x^2 + 2x + 3x + 6$ Then, take the second term in the first binomial and distribute it through the second binomial

$= x^2 + 5x + 6$

<u>Example 2</u>

Multiply the binomials using FOIL.

$(x-8)(7x+4)$

$= 7x^2 + 4x - 56x - 32$

$= 7x^2 - 52x - 32$

<u>Factoring:</u>

Factoring is finding the things that multiply to make something else. 8 factors to 2·4. The answer to example 2 above, $7x^2 – 52x – 32$, factors to $(x - 8)(7x + 4)$, because $(x - 8)(7x + 4)$ multiplies to $7x^2 – 52x – 32$. There are several different types of factoring:

<u>Factoring Quadratic Trinomials (x^2+bx+c):</u>

When factoring quadratic trinomials with no coefficient in front of the x^2, ask yourself this question:

What two numbers multiply to give "c" and add to give "b"?

<u>Example 3</u>

Factor.

$x^2 + 7x + 12$

What two numbers multiply to give "12" and add to give "7"?

The answer to this question is 3 and 4. $3 \bullet 4 = 12$, and $3 + 4 = 7$

Therefore, the factors of $x^2 + 7x + 12$ are **$(x + 3)(x + 4)$.**

Example 4

Factor to SOLVE;

To solve an equation by factoring, you must manipulate the equation so that all the variables and the constants are in $x^2 + bx + c$ form on one side of the "=" and there is 0 on the other side.

$x^2 - 10x + 16 = 0$

What two numbers multiply to give "16" and add to give "-10"?

The answer to this question is -2 and -8.

Therefore, the factors of $x^2 - 10x + 16$ are $\mathbf{(x - 2)(x - 8)}$, so we have

$\mathbf{(x - 2)(x - 8) = 0}$

What value of x makes $(x - 2) = 0$? Answer, 2

What value of x makes $(x - 8) = 0$? Answer, 8 There are 2 solutions…x = 2 and x = 8

Example 5

Factor to solve.

$x^2 + 7x - 18 = 0$

What two numbers multiply to give "-18" and add to give "7"?

The answer to this question is 9 and -2.

Therefore, the factors of $x^2 + 7x - 18$ are $\mathbf{(x + 9)(x - 2)}$, so we have

$\mathbf{(x + 9)(x - 2) = 0}$

What value of x makes $(x + 9) = 0$? Answer, - 9

What value of x makes $(x - 2) = 0$? Answer, 2 There are 2 solutions…x = - 9 and x = 2

Try These:

Simplify.

1. $(x^2)(3x) =$
2. $\frac{10x^5}{2x^2} =$
3. $(4x)(8x) =$

Distribute (or FOIL) to simplify.

4. $3x(x + 8)$
5. $-2(3x^2 - 4x + 6)$
6. $(x + 4)(x - 10)$

Factor each expression.

7. $x^2 - 3x - 28$
8. $x^2 + 8x - 48$

Solve each quadratic by factoring.

9. $x^2 + 12x + 11 = 0$
10. $x^2 + 5x - 14 = 0$

Try These Answers:

1. $3x^3$ 2. $5x^3$ 3. $32x^2$ 4. $3x^2 + 24x$ 5. $-6x^2 + 8x - 12$

6. $x^2 - 6x - 40$ 7. $(x-7)(x+4)$ 8. $(x+12)(x-4)$ 9. $x = -11$ or -1

10. x = -7, x = 2

The Quadratic Formula:

The Quadratic Formula is the only method that can be used to solve any quadratic equation. To find the value of "x" in a quadratic of the form $ax^2 + bx + c = 0$, use the following (if it cannot be factored):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 6

Solve using the quadratic formula.

$x^2 = 2x + 3$ We need to rewrite this to put it in the form $ax^2 + bx + c = 0$.

$$x^2 - 2x - 3 = 0$$

$$a = 1$$

$$b = \text{-}2$$

$$c = \text{-}3$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - (-12)}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$x = \frac{2 \pm 4}{2}$$

$$x = \frac{2+4}{2} \qquad \textit{and} \qquad x = \frac{2-4}{2}$$

$$x = 3 \qquad\qquad x = -1$$

***Some important notes about squares and square roots**

~ squaring numbers

$(-5)^2 = (-5)(-5) = 25$

$5^2 = 5 \bullet 5 = 25$ so if $x = \sqrt{25}$ then, $x = \pm 5$ (x could be 5 or -5)

$\sqrt{-25} = \textit{undefined}$ you CANNOT find the square root of a negative number!

~ squaring numbers and variables

$-2x^2$ means $-2 \bullet x^2$ which equals $(-2)(x)(x) = \mathbf{-2x^2}$

$(-2x)^2$ means $(-2x) \bullet (-2x)$ which equals $(-2)(-2)(x)(x) = \mathbf{4x^2}$

Try these:

Solve using the quadratic formula:

1. $-2x^2 + 6x + 9 = 0$ 2. $4x^2 - 13x + 3 = 0$ 3. $x^2 + 7x - 2 = 0$

Simplify:

4. $(-3x)^2$ 5. $(2x^2)(-4x)$ 6. $\sqrt{-64}$ 7. $x = \sqrt{81}$

Try these answers:

1. $x = \frac{3 \pm 3\sqrt{3}}{2}$ 2. $x = \frac{1}{4}, x = 3$ 3. $x = \frac{-7 \pm \sqrt{57}}{2}$

4. $9x^2$ 5. $-8x^3$ 6. Undefined 7. x = 9 or -9

H. Solving Systems of Equations

There are many ways to solve systems of equations (two or more equations with the same variables). The solution to the system is the ONE point on the graph where the 2 points intersect, that is the one combination of (x, y) that will make BOTH equations true. We will focus on solving by substitution.

Solving by Substitution:

To solve a system by substitution, you need to solve one of the equations for one of the variables. Then, you substitute what you get for the variable into the OTHER equation. Then, solve for the remaining variable. Finally, substitute the value of the first variable into either equation and solve for the other variable.

Example 1

Solve the system of equations using substitution.

$y = 2x$ Here, the 1st equation is already solved for y. We know 'y' equals ' 2x', so substitute '2x' in for 'y' in the 2nd equation.

$3x + y = 35$

Substituting, the 2nd equation becomes $3x + 2x = 35$

Combine like terms to simplify $5x = 35$

Isolate x (divide both sides by 5) $x = 7$

Now , we know 'x', we need to find 'y' so

Substitute '7' in for 'x' in the 1st equation

$y = 2x$

$y = 2(7)$

$y = 14$

We now know x = 7 and y = 14, so the solution to this system of equations is the point (7 , 14)

Example 2

Solve the system of equations using substitution.

$y = 2x + 3$

$y = -2x - 1$

Here, both equations are already solved for y, so substitute (-2x-1) for y in the first equation.

$-2x - 1 = 2x + 3$

$-4 = 4x$

$-1 = x$

Now that we've solved for x, substitute What we got for x into either equation And solve for y.

$y = 2x + 3$

$y = 2(-1) + 3$

$y = 1$

So, the solution to this system of equations is *(-1, 1)*

Example 3

Solve using substitution.

$$\begin{aligned} y - 2x &= 3 \\ y + 2x &= -1 \end{aligned}$$

You must solve ONE of the equations for one variable first.

y – 2x = 3 (solving the 1st equation for y)

+2x +2x

y = 2x + 3 Now substitute (2x + 3) in to the 2nd equation for y.

y + 2x = -1 (original 2nd equation)

(2x + 3) + 2x = -1

4x + 3 = - 1

- 3 - 3

$\frac{4x}{4} = \frac{-4}{4}$

x = - 1 substitute "-1" in for "x" into the original 1st equation

y – 2x = 3

y – 2(-1) = 3

y + 2 = 3

-2 -2

y = 1 so, since x = -1, and y = 1, the solution to this system is the point (-1, 1)

Try These:

Solve each of the following systems using substitution.

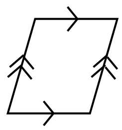
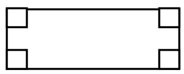
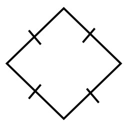
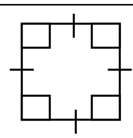
1. $\begin{aligned} y &= 2x + 3 \\ y &= 5x + 9 \end{aligned}$
2. $\begin{aligned} y &= 5 - x \\ x - y &= 7 \end{aligned}$
3. $\begin{aligned} x - 2y &= 8 \\ -x + 3y &= -15 \end{aligned}$
4. y – 2x = 3
   y + 2x = -1

Try These Answers:

1. *(-2, -1)*
2. *(6, -1)*
3. *(-6, -7)*
4. *(-1, 1)*

I. Geometry Vocabulary

Over the years you have been taught various Geometry vocabulary which will be useful to remember coming into this year.

Term	Drawing	Definition and comment.
Parallelogram		A parallelogram is a quadrilateral with two pairs of parallel sides.
Rectangle		A rectangle is a quadrilateral with four right angles. Note: The definition has nothing to do with congruent sides.
Rhombus		A rhombus is a quadrilateral with four congruent sides.
Square		A square is a quadrilateral with four congruent sides and four right angles.

There are two sets of terms for describing triangles.

Names based on the size of angles:

- An acute triangle is a triangle with angles that all measure less than 90^o.
- An obtuse triangle is a triangle with one angle that measures greater than 90^o.
- A right triangle is a triangle with one angle that measures exactly 90^o.

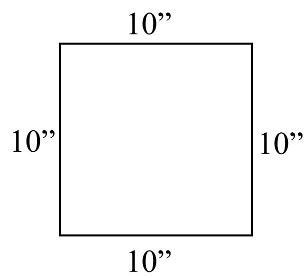
Names based on the size of sides:

- A scalene triangle is a triangle whose sides have three different lengths.
- An isosceles triangle is a triangle with at least one pair of sides that are the same length.
- An equilateral triangle is a triangle whose sides are all the same length.

Formulas you have learned:

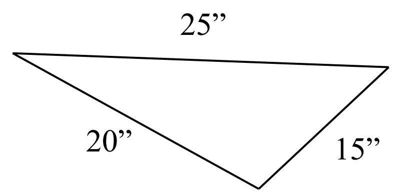
Area of a rectangle = bh   Perimeter of a rectangle = 2b + 2h
Area of a triangle = ½ bh
Area of a circle = πr^2 Circumference of a circle = $2\pi r$
The radius of circle = ½ the diameter of the circle.

Example 1:
Identify the figure shown



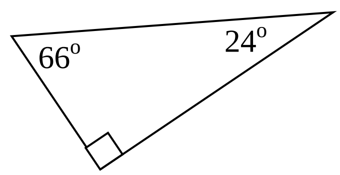
This is rhombus because all that is known is that it has four congruent sides.

Example 2:
Identify the figure shown



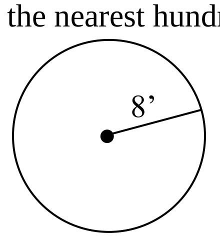
This is a scalene triangle because none of the sides are congruent.

Example 3:
Identify the figure shown.



This is a right triangle because it has a right angle.

Example 4:
Find the area of the figure shown, answer to the nearest hundredth.



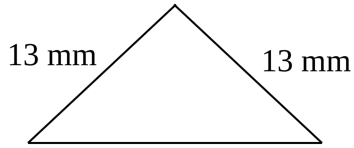
$A = \pi r^2 = \pi 8^2 = 64\pi = 201.0619298\ldots..$
$A = 201.06\ ft^2$

Note: Do not use 3.14 for π unless you are being made to do so. Π does not equal 3.14. 3.14 is an approximation for π, and a bad one at that. If your calculator has a π key, use the π key. If you must approximate π, use at least 3.1416. People will ask you to use 3.14 to simplify correcting, so you will sometimes be forced to use it; deeply regret these events with your entire heart and soul.

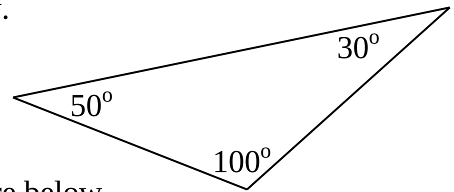
Try these:

1. Identify the figure to the right.

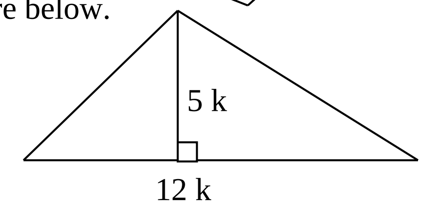
2. Identify the figure below.



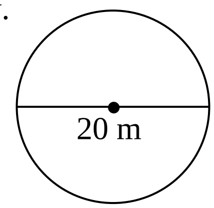
3. Identify the figure below.



4. Find the area of the figure below.



5. Find the circumference of the figure below.



Answers to try these:

1. A rectangle 2. An isosceles triangle 3. An obtuse triangle 4. 30 k^2
5. 20 π or 62.83 m