Curriculum Development
In the Fairfield Public Schools

FAIRFIELD PUBLIC SCHOOLS
FAIRFIELD, CONNECTICUT

Multivariable Calculus 61

Board of Education Approved 03/27/2007

MULTIVARIABLE CALCULUS 61

Statement of Purpose

Multivariable Calculus 61 is the culmination of the Fairfield Public Schools mathematics program for our most capable and committed mathematics students. It is designed to provide a course that is comparable to a college or university third semester Calculus course.

Audience

Students who have excelled in our most rigorous mathematics sequence and have completed BC Calculus will make up the Multivariable Calculus class.

Prerequisites

Calculus BC and Teacher Recommendation

Course Description

Multivariable Calculus is a rigorous second year course in college level calculus. This course provides an in-depth study of vectors and the calculus of several variables for the student who has successfully completed Calculus BC. The successful student will bring to the course a solid understanding of the concepts of first-year calculus as well as the ability to approach complex problems and applications with insight, imagination, and persistence. Major topics will include vector operations and analysis, functions of two or more variables and their partial derivatives, multiple integration.

Course Objectives

Students will be able to:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- recognize vectors as systems that have some, but not all, of the properties of real numbers.
- recognize the effect of changes in parameters on the graphs of multivariable functions or relations.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and symmetries for multivariable functions.
- recognize that the slope of the tangent line to the surface represents the rate of change in a particular direction.
- understand and use optimization strategies including maximums and minimums.
- apply the concepts of limits and asymptotic behavior of multivariable functions.
- approximate methods to solve measurement problems using multivariable functions.

- examine the effects of transformations on multivariable functions.

Skill Objectives

Students will:

- graph points in planes in three dimensions.
- use the distance formula.
- describe and sketch regions involving spheres.
- describe vectors algebraically, graphically, and verbally.
- compute with vectors using properties of vectors and vector operations.
- solve problems involving force and velocity using components of vectors.
- compute dot product of vectors.
- use dot product to determine the angle between vectors.
- find scalar and vector projections.
- compute and use cross products.
- compute and use scalar and vector triple products.
- compute and use vector and parametric equations of lines.
- compute and use equations of planes.
- sketch graphs and describe properties of functions in two variables.
- work with quadric surfaces.
- convert from one coordinate system to another.
- graph functions given in spherical and cylindrical coordinates.
- sketch and recognize graphs of space curves given parametrically.
- find the domain, range and limits of vector functions.
- compute derivatives of vector functions.
- determine tangent vectors of space curves.
- compute integrals of vector functions.
- compute arc length of space curves.
- parameterize curves with respect to arc length.
- compute the curvature of a curve.
- compute the normal and binormal vectors and the associated planes of a curve.
- compute the velocity and acceleration of a particle.
- solve problems involving motion, acceleration or force.
- graph and recognize parametric surfaces.
- find parametric representations of surfaces.
- find domain and range of functions of two variables.
- describe the level surfaces of functions with three variables.
- determine if a function f(x, y), has a limit at (a, b).
- determine the points of continuity of a function f(x, y).
- compute and interpret partial derivatives.
- use Clairault's Theorem to compute higher partial derivatives.
- verify whether or not a given function satisfies a partial differential equation.
- compute the tangent plane to a surface given by a function of two variables.
- determine if a function is differentiable.
- use linearization to approximate values of a function.

- compute tangent planes to parametric surfaces.
- compute derivatives using the Chain Rule.
- use implicit differentiation to compute derivatives.
- compute directional derivatives.
- find and apply the gradient vector.
- find tangent planes and normal lines to level surfaces.
- find local maximum and minimum values.
- compute absolute maximum and minimum values of a function.
- determine the saddle points of a function, if any.
- use the method of Lagrange multipliers to determine extreme values of a function subject to constraints.
- use a double Reimann Sum to approximate double integrals.
- evaluate double integrals by computing volumes.
- evaluate double integrals over general regions.
- evaluate double integrals over polar regions.
- use double integrals to compute mass.
- compute moments of inertia and centers of mass.
- compute triple integrals.
- apply Fubini's Theorem.
- apply triple integrals to problems of volume, density and mass.
- compute triple integrals using cylindrical coordinates.
- compute triple integrals using spherical coordinates.
- find the image of a set under a transformation.
- compute the Jacobian of a transformation.
- use change of variables to simplify and evaluate multiple integrals.

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.

Extended

1.1 Students should understand and describe patterns and functional relationships.

1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.

1.2 Students should represent and analyze quantitative relationships in a variety of ways.

1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.

Extended

2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.

2.2a Students should investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
Extended
3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.
3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.
3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.
3.3a Students should approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is a vector?
- What operations can be performed using vectors?
- How can you use vectors to determine equations of lines and planes in space?
- What is a vector function?
- What is a derivative/integral of a vector function?
- What is curvature?
- What is a parametric surface?
- What is a partial derivative?
- What is a directional derivative?
- What is a gradient vector?
- What is a LaGrange Multiplier?
- What is an iterated integral?
- What is a Jacobian?

Units Of Study

Unit 1: Vectors and the Geometry of Space

Math Standards

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Extended
1.1 Students should understand and describe patterns and functional relationships.
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1.2 Students should represent and analyze quantitative relationships in a variety of ways.
1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
Extended
2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.
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Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
Extended
3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.
3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is a vector?
- What operations can be performed using vectors?
- How can you use vectors to determine equations of lines and planes in space?

Core Topics

- Vectors
- Dot product
- Cross product
- Equations of lines and planes in space
- Functions and surfaces in space
- Cylindrical and spherical coordinates

Unit Objectives

Students will:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- recognize the effect of changes in parameters on the graphs of multivariable functions or relations.
- recognize vectors as systems that have some, but not all, of the properties of real numbers.

Skill Objectives

Students will:

- graph points in planes in three dimensions.
- use the distance formula.
- describe and sketch regions involving spheres.
- describe vectors algebraically, graphically, and verbally.
- compute with vectors using properties of vectors and vector operations.
- solve problems involving force and velocity using components of vectors.
- compute dot product of vectors.
- use dot product to determine the angle between vectors.
- find scalar and vector projections.
- compute and use cross products.
- compute and use scalar and vector triple products.
- compute and use vector and parametric equations of lines.
- compute and use equations of planes.
- sketch graphs and describe properties of functions in two variables.
- work with quadric surfaces.
- convert from one coordinate system to another.
- graph functions given in spherical and cylindrical coordinates.

Sample Assessment

Hanging a Sign

Pacing

10 weeks

Unit 2: Vector Functions

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
Extended
1.1 Students should understand and describe patterns and functional relationships.
1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.
1.2 Students should represent and analyze quantitative relationships in a variety of ways.
1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Numerical and Proportional Reasoning – Quantitative relationships can be expressed numerically in multiple ways in order to make connections and simplify calculations using a variety of strategies, tools and technologies.
Extended
2.2 Students should use numbers and their properties to compute flexibly and fluently, and to reasonably estimate measures and quantities.
2.2a Students should investigate mathematical properties and operations related to objects that are not numbers.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
Extended
3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.
3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How are quantitative relationships represented by numbers?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is a vector function?
- What is a derivative/integral of a vector function?
- What is curvature?
- What is a parametric surface?

Core Topics

- Vector functions and space curves

- Derivatives and integrals of vector functions
- Arc length and curvature
- Velocity and acceleration
- Parametric surfaces

Unit Objective

Students will:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and symmetries for multivariable functions.
- recognize vectors as systems that have some, but not all, of the properties of real numbers.

Skill Objectives

Students will:

- sketch and recognize graphs of space curves given parametrically.
- find the domain, range and limits of vector functions.
- compute derivatives of vector functions.
- determine tangent vectors of space curves.
- compute integrals of vector functions.
- compute arc length of space curves.
- parameterize curves with respect to arc length.
- compute the curvature of a curve.
- compute the normal and binormal vectors and the associated planes of a curve.
- compute the velocity and acceleration of a particle.
- solve problems involving motion, acceleration or force.
- graph and recognize parametric surfaces.
- find parametric representations of surfaces.

Sample Assessment

Fireworks Display

Pacing

8 weeks

Unit 3: Partial Derivatives

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
Extended
1.1 Students should understand and describe patterns and functional relationships.
1.1a Students should model real world situations and make generalizations about mathematical relationships using a variety of patterns and functions.
1.2 Students should represent and analyze quantitative relationships in a variety of ways.
1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
Extended
3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.
3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.
3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.
3.3a Students should approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is a partial derivative?
- What is a directional derivative?
- What is a gradient vector?
- What is a LaGrange Multiplier?

Core Topics

- Functions of several variables
- Limits and continuity
- Partial derivatives
- Tangent planes and linear approximations
- Chain rule
- Directional derivatives and gradient
- Extrema and saddle points

- LaGrange multiplier

Unit Objectives

Students will

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- approximate methods to solve measurement problems using multivariable functions.
- understand and use optimization strategies including maximums and minimums.
- apply the concepts of limits and asymptotic behavior of multivariable functions.
- relate the graphical representation of a function to its function family and find equations, intercepts, maximum or minimum values, asymptotes and symmetries for multivariable functions.
- recognize that the slope of the tangent line to the surface represents the rate of change in a particular direction.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.

Skill Objectives

Students will:

- find domain and range of functions of two variables.
- describe the level surfaces of functions with three variables.
- determine if a function f(x,y), has a limit at (a,b).
- determine the points of continuity of a function f(x,y).
- compute and interpret partial derivatives.
- use Clairault's Theorem to compute higher partial derivatives.
- verify whether or not a given function satisfies a partial differential equation.
- compute the tangent plane to a surface given by a function of two variables.
- determine if a function is differentiable.
- use linearization to approximate values of a function.
- compute tangent planes to parametric surfaces.
- compute derivatives using the Chain Rule.
- use implicit differentiation to compute derivatives.
- compute directional derivatives.
- find an apply the gradient vector.
- find tangent planes and normal lines to level surfaces.
- find local maximum and minimum values.
- compute absolute maximum and minimum values of a function.
- determine the saddle points of a function, if any.
- use the method of Lagrange multipliers to determine extreme values of a function subject to constraints.

Sample Assessment
Wind Chill Effect

Pacing
10 weeks

Unit 4: Multiple Integrals

Math Standards

Algebraic Reasoning: Patterns And Functions – Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies.
Extended
1.1 Students should understand and describe patterns and functional relationships.
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1.2 Students should represent and analyze quantitative relationships in a variety of ways.
1.2a Students should relate the behavior of functions and relations to specific parameters and determine functions to model real world situations.

Geometry and Measurement – Shapes and structures can be analyzed, visualized, measured and transformed using a variety of strategies, tools and technologies.
Extended
3.2 Students should use spatial reasoning, location and geometric relationships to solve problems.
3.2a Students should use a variety of coordinate systems and transformations to solve geometric problems in two- and three-dimensions using appropriate tools and technology.
3.3 Students should develop and apply units, systems, formulas and appropriate tools to estimate and measure.
3.3a Students should approximate measurements that cannot be directly determined with some degree of precision using appropriate tools, techniques and strategies.

Essential Questions

- How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?
- How do geometric relationships and measurements help us to solve problems and make sense of our world?

Focus Questions

- What is an iterated integral?
- What is a Jacobian?

Core Topics

- Double integral over rectangles and general regions
- Double integrals in polar coordinates
- Applications of double integrals
- Triple integrals
- Triple integrals in spherical/cylindrical coordinates
- Change of variable technique with multiple integrals

Unit Objectives

Students will be able to:

- analyze essential relations in a problem to determine possible functions that could model the situation.
- explore the graphs of multivariable functions.
- visualize three-dimensional objects from different perspectives and analyze cross-section and volume.
- use Cartesian, polar, cylindrical, and spherical systems to represent and analyze multivariable functions.
- recognize the effect of changes in parameters on the graphs of multivariable functions or relations.
- examine the effects of transformations on multivariable functions.
- approximate methods to solve measurement problems using multivariable functions.

Skill Objectives

Students will:

- use a double Reimann Sum to approximate double integrals.
- evaluate double integrals by computing volumes.
- evaluate double integrals over general regions.
- evaluate double integrals over polar regions.
- use double integrals to compute mass.
- compute moments of inertia and centers of mass.
- compute triple integrals.
- apply Fubini's Theorem.
- apply triple integrals to problems of volume, density and mass.
- compute triple integrals using cylindrical coordinates.
- compute triple integrals using spherical coordinates.
- find the image of a set under a transformation.
- compute the Jacobian of a transformation.
- use change of variables to simplify and evaluate multiple integrals.

Sample Assessment

Swimming Pool

Pacing

10 weeks