

# BRUSHING UP on ESSENTIAL ALGEBRA SKILLS

To Get you Ready to

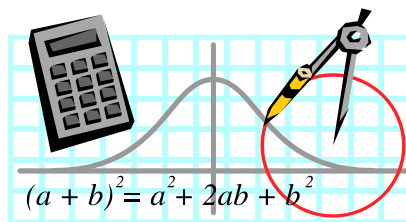


# PRE- CALCULUS

Name: \_\_\_\_\_

Directions: Use pencil and the space provided next to the question to show all work. The purpose of this packet is to give you a review of basic skills. Please refrain from using a calculator!

Prepared by:  
Mrs. Trebat  
Fairfield Ludlowe High School  
mtrebat@fairfield.k12.ct.us



# BRUSHING UP ON BASIC ALGEBRA SKILLS

Mrs. Trebat

Name: \_\_\_\_\_ DUE DATE: \_\_\_\_\_

**Directions:** Use pencil, show work, box in your answers.

## Monomial Factors of Polynomials

A **monomial** is an expression that is either a numeral, a variable, or the product of a numeral and one or more variables. Example of monomials:  $7$ ,  $x$ ,  $6x^2y^3$ .

A sum of monomials is called a **polynomial**. Some polynomials have special names:

**Binomials** (two terms):  $3x - 5$  or  $2xy + x^2$

**Trinomials** (3 terms):  $x^2 + 5x - 15$  or  $x^2 - 6xy + 9y^2$

Divide:

1)  $\frac{24x - 12}{6}$

2)  $\frac{8x^4 - 4x^3 - 6x^2}{-2x^2}$

Factor (monomials only): Example:  $15a - 25b + 35 = 5(3a - 5b + 7)$

3)  $7a^3 - 21a^2 - 14a$

4)  $5ax^2 - 10a^2x + 15a^3$

Simplify: Example:

$$\frac{14x - 21}{7} - \frac{10x - 25}{5} = \frac{7(2x - 3)}{7} - \frac{5(2x - 5)}{5} = 2x - 3 - 2x + 5 = 2$$

Factor                      Cancel out common factors  
Simplify and combine

5)  $\frac{6a + 9b}{3} - \frac{7a + 21b}{7} =$

6)  $\frac{x^2y - 3x^2y^2}{xy} + \frac{6xy + 9xy^2}{3y} =$

## Multiplying Binomials Mentally

Write each product as a trinomial:

7)  $(x - 9)(x + 4) =$

8)  $(4 - x)(1 - x) =$

9)  $(2a + 5)(a - 2) =$

10)  $(2x - 5)(3x + 4) =$

Find the values of p, q, and r that make the equation true.

11)  $(px + q)(2x + 5) = 6x^2 + 11x + r$

## Difference of Two Squares

You must use the shortcut below (do not "FOIL"!!!!)

$$(a + b)(a - b) = a^2 - b^2$$
$$(First + Second) \times (First - Second) = (First)^2 - (Second)^2$$

Write each product as a binomial:

12)  $(x + 7)(x - 7) =$

13)  $(y + 8)(y - 8) =$

14)  $(5x + 2)(5x - 2) =$

15)  $(8x - 11)(8x + 11) =$

16)  $(4a + 5b)(4a - 5b) =$

17)  $(x^2 - 9y)(x^2 + 9y) =$

Now let's try reversing the process above... Factor:

18)  $x^2 - 36 =$

19)  $m^2 - 81 =$

20)  $25a^2 - 1 =$

21)  $49x^2 - 9y^2 =$

Factor each expression as the difference of two squares. Then simplify.

**Example:**  $x^2 - (x - 3)^2 = [x - (x - 3)][x + (x - 3)] = 3(2x - 3)$

Apply the formula

simplify

22)  $(x + 4)^2 - x^2 =$

23)  $9(x + 1)^2 - 4(x - 1)^2 =$

## Squares of Binomials and Perfect Square Trinomials

Every time you square a binomial, the same pattern comes up. To speed up the process, we should memorize:

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{and} \quad (a - b)^2 = a^2 - 2ab + b^2$$

A trinomial is called a perfect square trinomial if it is the square of a binomial. For example,  $x^2 - 6x + 9$  is a perfect square trinomial because it is equal to  $(x - 3)^2$ .

Write each square as a trinomial.

24)  $(a - 9)^2 =$

25)  $(x + 7)^2 =$

26)  $(4x - 1)^2 =$

27)  $(5a - 2b)^2 =$

Decide if each trinomial is a perfect square. If it is, factor it. Otherwise, write **not a perfect square**.

28)  $a^2 + 6a + 9 =$

29)  $y^2 - 14y + 49 =$

30)  $121 - 22x + x^2 =$

31)  $9a^2 + 30ab + 100b^2 =$

32)  $49x^2 - 28xy + 4y^2 =$

33)  $25x^2 - 15xy + 36y^2 =$

34)  $a^2b^2 - 12ab + 36 =$

35)  $121 - 33x^2 + 9x^4$

36) Show that  $a^4 - 8a^2 + 16$  can be factored as  $(a + 2)^2(a - 2)^2$ .

37) Solve and check:  $(x + 2)^2 - (x - 3)^2 = 35$

## Factoring Quadratic Trinomials

To factor a trinomial of the form  $x^2 + bx + c$ , you must find two numbers,  $r$  and  $s$ , whose product is  $c$  and whose sum is  $b$ .

$$x^2 + bx + c = (x + r)(x + s)$$

When you find the product  $(x + r)(x + s)$  you obtain

$$x^2 + bx + c = x^2 + (r + s)x + rs$$

Example: Factor  $x^2 - 2x - 15$ .

- List the factors of -15 (the last term).
- Either write them down or do this mentally. Find the pairs of factors with sum -2 (the middle term).

Factors of -15	Sum of the factors
1, -15	-15 ( <i>discard</i> )
-3, 5	2 ( <i>discard</i> )
3, -5	-2 ( <i>keep</i> )

$$\therefore x^2 - 2x - 15 = (x - 5)(x + 3)$$

Check the result by multiplying...

Factor. Check by multiplying (mentally).

38.  $x^2 + 8x + 12 =$  \_\_\_\_\_

39.  $x^2 - 7x + 12 =$  \_\_\_\_\_

40.  $x^2 - 4x + 12 =$  \_\_\_\_\_

41.  $x^2 - 9x + 18 =$  \_\_\_\_\_

42.  $x^2 - 3x - 18 =$  \_\_\_\_\_

43.  $x^2 + 11x + 18 =$  \_\_\_\_\_

44.  $x^2 - 5x - 36 =$  \_\_\_\_\_

45.  $x^2 - 15x + 36 =$  \_\_\_\_\_

46.  $x^2 - 9x - 36 =$  \_\_\_\_\_

47.  $x^2 + 3x - 28 =$  \_\_\_\_\_

**Factoring General Quadratic Trinomials of the type  $ax^2 + bx + c$**

Example: Factor  $2x^2 + 7x - 9 = (2x + 9)(x - 1)$

↑                      ↑  
List all factors    List factors of -9  
Of  $2x^2$

Factor:

48)  $3x^2 + 7x + 2$

49)  $2x^2 + 5x + 3$

50)  $2x^2 - 15x + 7$

51)  $3a^2 + 4a - 4$

52)  $5a^2 - 6a - 2$

53)  $3x^2 - 2x - 5$

54)  $3m^2 + 7m - 6$

55)  $4a^2 - a - 3$

**Factor by Grouping**

Example 1:  $7(a - 2) + 3a(a - 2) = (7 + 3a)(a - 2)$

Notice that we "**factored out**" the common factor  $(a - 2)$

Example 2:  $5(x - 3) - 2x(3 - x)$

Notice that  $x - 3$  and  $3 - x$  are opposites.

$5(x - 3) - 2x(3 - x) = 5(x - 3) + 2x(x - 3) = (5 + 2x)(x - 3)$

56)  $2x(x - y) + y(y - x)$

57)  $3a(2b - a) - 2b(a - 2b)$

Group and Factor:

58)  $3a + ab + 3c + bc$

59)  $3a^3 + a^2 + 6a + 2$

60)  $3ab - b - 4 + 12a$

## Solving Equations by Factoring - The Zero Product Property

Key Concept:

### **Zero-Product Property**

For all real numbers  $a$  and  $b$ :

$$a \cdot b = 0 \text{ if and only if } a = 0 \text{ or } b = 0$$

A product of factors is equal to zero if and only if at least one of the factors is 0.

Solve:

61)  $(x + 5)(x - 3) = 0$

62)  $2x(x - 9) = 0$

63)  $(2a - 3)(3a + 2) = 0$

64)  $3x(5x + 2)(x - 7) = 0$

65)  $a^2 - 3a + 2 = 0$

66)  $x^2 - 12x + 35 = 0$

67)  $b^2 = 4b + 32$

68)  $25x^2 - 16 = 0$

69)  $7x^2 = 18x - 11$

70)  $8y^3 - 2y^2 = 0$

71)  $4x^3 - 12x^2 + 8x = 0$

72)  $9x^3 + 25x = 30x^2$

Sample for items 73-74:  $(a - 1)(a + 3) = 12$

$$a^2 + 2a - 3 - 12 = 0 \quad \text{expand the left side; bring over 12; set it = 0;}$$

$$(a - 3)(a + 5) = 0 \quad \text{combine like terms; factor it;}$$

$$a = 3 \text{ or } a = -5$$

73)  $(x + 1)(x - 5) = 16$

74)  $(2z - 5)(z - 1) = 2$

**Simplifying Fractions** - Follow the 2 examples below.

**Example 1:** Simplify  $\frac{3x+6}{3x+3y}$ .

**Solution:**  $\frac{3x+6}{3x+3y} = \frac{3(x+2)}{3(x+y)}$  Factor the numerator and denominator; look for common factors;  
 $= \frac{x+2}{x+y}, x \neq -y$  Cancel out common factor which is 3;

**Example 2:** Simplify  $\frac{x^2-9}{(2x+1)(3-x)}$

**Solution:**  $\frac{x^2-9}{(2x+1)(3-x)} = \frac{(x+3)(x-3)}{-(2x+1)(x-3)}$  First factor the numerator; "pull out" a negative in the factor (3-x) to make it a common factor with the numerator;  
 $= -\frac{x+3}{2x+1}, \left(x \neq -\frac{1}{2}, x \neq 3\right)$  exclude the first as it would make the denominator =0; exclude the 2<sup>nd</sup> as it would make both numerator and denominator =0.

Simplify. Give any restrictions on the variable.

75)  $\frac{5x-10}{x-2}$

76)  $\frac{2a-4}{a^2-4}$

77)  $\frac{(x-5)(2+7x)}{(5-x)(7x+2)}$

78)  $\frac{x^2+8x+16}{16-x^2}$



## Multiplying Fractions

### **Multiplication Rule for Fractions**

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \quad \text{To multiply fractions, you multiply their numerators and multiply their denominators.}$$

Note: You can multiply first and then simplify, or you can simplify first and then multiply.

$$\begin{aligned} \text{Example: } \frac{x^2 - x - 12}{x^2 - 5x} \cdot \frac{x^2 - 25}{x + 3} &= \frac{(x-4)\cancel{(x+3)}}{x\cancel{(x-5)}} \cdot \frac{\cancel{(x-5)}(x+5)}{\cancel{x+3}} \\ &= \frac{(x-4)(x+5)}{x} \quad \text{Notice how common factors were cancelled.} \end{aligned}$$

Simplify.

$$79) \frac{a+2}{a^2} \cdot \frac{3a}{a^2-4}$$

$$80) \frac{a^2 - x^2}{a^2} \cdot \frac{a}{3x - 3a}$$

81) A triangle has base  $\frac{3x}{4}$  cm and height  $\frac{8}{9x}$  cm. What is its area?

## Dividing Fractions

$$\text{Division Rule for Fractions: } \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \quad \text{Example: } \frac{7}{9} \div \frac{2}{3} = \frac{7}{9} \cdot \frac{3}{2} = \frac{7}{6}$$

*To divide by a fraction, you multiply by its reciprocal!*

$$82) \frac{2+2a}{6} \div \frac{1+a}{9} =$$

$$83) \frac{x^2-1}{2} \div \frac{x+1}{16} =$$

$$84) \frac{2a+2b}{a^2} \div \frac{a^2-b^2}{4a}$$

$$85) \frac{4x^2-25}{x^2-16} \div \frac{12x+30}{2x^2+8x} =$$

### Adding and Subtracting Algebraic Fractions

- Key Steps:**
- (1) Find the Least Common Denominator (**LCD**);
  - (2) Re-write each fraction being added or subtracted with the same common denominator.
  - (3) Add or subtract their numerators and write the result over the common denominator.

Example:

$$\begin{aligned} \frac{3}{6x-30} + \frac{8}{9x-45} &= \frac{3}{6(x-5)} + \frac{8}{9(x-5)} && \text{Factor out denominators to more easily identify LCD} \\ &= \frac{9}{18(x-5)} + \frac{16}{18(x-5)} && \text{multiply the first fraction by 3, the second by 2} \\ &= \frac{25}{18(x-5)} \text{ or } \frac{25}{18x-90} \end{aligned}$$

$$86) \frac{4x+3}{3} - \frac{7x}{4} + \frac{x-3}{6} =$$

$$87) \frac{2}{x-3} + \frac{4}{x+3} =$$

$$88) \frac{x}{x^2-1} + \frac{4}{x+1} =$$

$$89) \frac{3a}{a-2b} + \frac{6b}{2b-a} =$$

$$90) \frac{x-11}{x^2-9} - \frac{x-7}{x^2-3x} =$$

**BRUSHING UP ON BASIC ALGEBRA SKILLS - Solved Problems**

Mrs. Trebat

Name: \_\_\_\_\_

**Directions:** Use pencil. You must show work for credit. Your final answers must be clearly identified.

**Monomial Factors of Polynomials**

A monomial is an expression that is either a numeral, a variable, or the product of a numeral and one or more variables. Example of monomials: 7, x, 6x<sup>2</sup>y<sup>3</sup>.

A sum of monomials is called a polynomial. Some polynomials have special names:

**Binomials** (two terms): 3x-5 or 2xy+x<sup>2</sup>

**Trinomials** (3 terms): x<sup>2</sup>+5x-15 or x<sup>2</sup>-6xy+9y<sup>2</sup>

Divide:

$$1) \frac{24x-12}{6} = \frac{2(2x-1)}{6} = \frac{2x-1}{3} = 4x-2$$

$$2) \frac{8x^4-4x^3-6x^2}{-2x^2} = \frac{2x^2(4x^2-2x-3)}{-2x^2} = -4x^2+2x+3$$

**Factor:** Example: 15a-25b+35=5(3a-5b+7)

$$3) \frac{7a^3-21a^2-14a}{7a(a^2-3a-2)}$$

$$4) \frac{5ax^2-10a^2x+15a^3}{5a(x^2-2ax+3a^2)}$$

**Simplify:** Example:

$$\frac{14x-21}{7} - \frac{10x-25}{5} = \frac{7(2x-3)}{7} - \frac{5(2x-5)}{5} = 2x-3-2x+5=2$$

Factor                      Cancel out common factors  
Simplify and combine

$$5) \frac{6a+9b}{3} - \frac{7a+21b}{7} = \frac{3(2a+3b)}{3} - \frac{7(a+3b)}{7} = 2a+3b-a-3b = a$$

$$6) \frac{x^2y-3x^2y^2}{xy} + \frac{6xy+9xy^2}{3y} = \frac{x(x-3xy)}{xy} + \frac{3xy(2+3y)}{3y} = x-3xy+2x+3xy = 3x$$

**Multiplying Binomials Mentally**

Write each product as a trinomial:

$$7) (x-9)(x+4) = x^2-5x-36$$

$$8) (4-x)(1-x) = 4-5x+x^2$$

$$9) (2a+5)(a-2) = 2a^2+a-10$$

$$10) (2x-5)(3x+4) = 6x^2-7x-20$$

Find the values of p, q, and r that make the equation true.

$$11) (px+q)(2x+5) = 6x^2+11x+r$$

$$2px^2+5px+2qx+5q = 6x^2+11x+r$$

EQUATING like Terms:  
 $2p=6 \Rightarrow p=3$      $5p+2q=11$      $5(3)=r$   
 $15+2q=11$      $\div 10=r$   
 $2q=-4$      $\div 2$   
 $q=-2$

**Difference of Two Squares**

You must use the shortcut below (do not "FOIL"!!!!)

$$(a+b)(a-b) = a^2 - b^2$$

$$(First + Second) \times (First - Second) = (First)^2 - (Second)^2$$

Write each product as a binomial:

$$12) (x+7)(x-7) = x^2-49$$

$$13) (y+8)(y-8) = y^2-64$$

$$14) (5x+2)(5x-2) = 25x^2-4$$

$$15) (8x-11)(8x+11) = 64x^2-121$$

$$16) (4a+5b)(4a-5b) = 16a^2-25b^2$$

$$17) (x^2-9y)(x^2+9y) = x^4-81y^2$$

Now let's try reversing the process above... Factor:

$$18) x^2-36 = (x+6)(x-6)$$

$$19) m^2-81 = (m+9)(m-9)$$

$$20) 25a^2-1 = (5a+1)(5a-1)$$

$$21) 49x^2-9y^2 = (7x+3y)(7x-3y)$$

Factor each expression as the difference of two squares. Then simplify.

**Example:**  $x^2 - (x-3)^2 = [x - (x-3)][x + (x-3)] = 3(2x-3)$

Apply the formula

simplify

$$22) (x+4)^2 - x^2 = (x+4+x)(x+4-x) = (2x+4)4 = 4(2x+4)$$

$$23) 9(x+1)^2 - 4(x-1)^2 = [3(x+1) + 2(x-1)][3(x+1) - 2(x-1)] = (3x+3+2x-2)(3x+3-2x+2) = (5x+1)(x+5)$$

### Squares of Binomials and Perfect Square Trinomials

Every time you square a binomial, the same pattern comes up. To speed up the process, we should memorize:

$$(a+b)^2 = a^2 + 2ab + b^2 \text{ and } (a-b)^2 = a^2 - 2ab + b^2$$

A trinomial is called a perfect square trinomial if it is the square of a binomial. For example,  $x^2 - 6x + 9$  is a perfect square trinomial because it is equal to  $(x-3)^2$ .

Write each square as a trinomial.

24)  $(a-9)^2 = a^2 - 18a + 81$

25)  $(x+7)^2 = x^2 + 14x + 49$

26)  $(4x-1)^2 = 16x^2 - 8x + 1$

27)  $(5a-2b)^2 = 25a^2 - 20ab + 4b^2$

Decide if each trinomial is a perfect square. If it is, factor it. Otherwise, write not a perfect square.

28)  $a^2 + 6a + 9 = (a+3)^2$

29)  $y^2 - 14y + 49 = (y-7)^2$

30)  $121 - 22x + x^2 = (11-x)^2$

31)  $9a^2 + 30ab + 100b^2 = \text{NOT A PERFECT SQUARE (due to middle term...)}$

32)  $49x^2 - 28xy + 4y^2 = (7x-2y)^2$

33)  $25x^2 - 15xy + 36y^2 = \text{NOT A PERFECT } \square !$

34)  $a^2b^2 - 12ab + 36 = (ab-6)^2$

35)  $121 - 33x^2 + 9x^4 = \text{NOT A PERFECT } \square !$

36) Show that  $a^4 - 8a^2 + 16$  can be factored as  $(a+2)^2(a-2)^2$ .

$$a^4 - 8a^2 + 16 = (a^2 - 4)^2 = [(a+2)(a-2)]^2 = (a+2)^2(a-2)^2$$

37) Solve and check:  $(x+2)^2 - (x-3)^2 = 35$

$$x^2 + 4x + 4 - (x^2 - 6x + 9) = 35$$

$$4x + 4 + 6x - 9 = 35$$

$$10x - 5 = 35$$

$$10x = 40$$

$$x = 4$$

### Factoring Quadratic Trinomials

To factor a trinomial of the form  $x^2 + bx + c$ , you must find two numbers,  $r$  and  $s$ , whose product is  $c$  and whose sum is  $b$ .

$$x^2 + bx + c = (x+r)(x+s)$$

When you find the product  $(x+r)(x+s)$  you obtain

$$x^2 + bx + c = x^2 + (r+s)x + rs$$

Example: Factor  $x^2 - 2x - 15$ .

- List the factors of -15 (the last term).
- Either write them down or do this mentally. Find the pairs of factors with sum -2 (the middle term).

Factors of -15	Sum of the factors
1, -15	-15 (discard)
-3, 5	2 (discard)
3, -5	-2 (keep)

$$\therefore x^2 - 2x - 15 = (x-5)(x+3)$$

Check the result by multiplying...

Factor. Check by multiplying (mentally).

38.  $x^2 + 8x + 12 = \underline{(x+6)(x+2)}$

39.  $x^2 - 7x + 12 = \underline{(x-4)(x-3)}$

40.  $x^2 - 4x + 12 = \underline{\text{NOT Factorable}}$

41.  $x^2 - 9x + 18 = \underline{(x-6)(x-3)}$

42.  $x^2 - 3x - 18 = \underline{(x-6)(x+3)}$

43.  $x^2 + 11x + 18 = \underline{(x+9)(x+2)}$

44.  $x^2 - 5x - 36 = \underline{(x-9)(x+4)}$

45.  $x^2 - 15x + 36 = \underline{(x-12)(x-3)}$

46.  $x^2 - 9x - 36 = \underline{(x-12)(x+3)}$

47.  $x^2 + 3x - 28 = \underline{(x+7)(x-4)}$

**Factoring General Quadratic Trinomials of the type  $ax^2 + bx + c$**

Example: Factor  $2x^2 + 7x - 9 = (2x+9)(x-1)$

List all factors of  $2x^2$       List factors of  $-9$

Factor:

- 48)  $3x^2 + 7x + 2 = (3x+1)(x+2)$
- 49)  $2x^2 + 5x + 3 = (2x+3)(x+1)$
- 50)  $2x^2 - 15x + 7 = (2x-1)(x-7)$
- 51)  $3a^2 + 4a - 4 = (3a-2)(a+2)$
- 52)  $5a^2 - 6a - 2 = (5a+1)(a-2)$
- 53)  $3x^2 - 2x - 5 = (3x-5)(x+1)$
- 54)  $3m^2 + 7m - 6 = (3m-2)(m+3)$
- 55)  $4a^2 - a - 3 = (4a+3)(a-1)$

**Factor by Grouping**

Example 1:  $7(a-2) + 3a(a-2) = (7+3a)(a-2)$

Notice that we factored out the common factor  $(a-2)$

Example 2:  $5(x-3) - 2x(3-x)$

Notice that  $x-3$  and  $3-x$  are opposites.

$5(x-3) - 2x(3-x) = 5(x-3) + 2x(x-3) = (5+2x)(x-3)$

56)  $2x(x-y) + y(y-x) = 2x(x-y) - y(x-y) = (x-y)(2x-y)$

57)  $3a(2b-a) - 2b(a-2b) = 3a(2b-a) + 2b(2b-a) = (2b-a)(3a+2b)$

**Group and Factor:**

58)  $3a+ab+3c+bc = (3a+a+b) + (3c+bc) = a(3+b) + c(3+b) = (a+c)(3+b)$

59)  $3a^3 + a^2 + 6a + 2 = (3a^3 + a^2) + (6a+2) = a^2(3a+1) + 2(3a+1) = (3a+1)(a^2+2)$

60)  $3ab - b - 4 + 12a = (3ab - b) - (4 - 12a) = b(3a-1) - 4(1-3a) = b(3a-1) + 4(3a-1) = (3a-1)(b+4)$

**Solving Equations by Factoring - The Zero Product Property**

Key Concept:

**Zero-Product Property**

For all real numbers  $a$  and  $b$ :

$a \cdot b = 0$  if and only if  $a = 0$  or  $b = 0$

A product of factors is equal to zero if and only if at least one of the factors is 0.

Solve:

- 61)  $(x+5)(x-3) = 0$   
 $x+5=0 \Rightarrow x=-5$  or  $x-3=0 \Rightarrow x=3$
- 62)  $2x(x-9) = 0$   
 $2x=0 \Rightarrow x=0$  or  $x-9=0 \Rightarrow x=9$
- 63)  $(2a-3)(3a+2) = 0$   
 $2a-3=0 \Rightarrow a=3/2$  or  $3a+2=0 \Rightarrow a=-2/3$
- 64)  $3x(5x+2)(x-7) = 0$   
 $3x=0 \Rightarrow x=0$  or  $5x+2=0 \Rightarrow x=-2/5$  or  $x-7=0 \Rightarrow x=7$
- 65)  $a^2 - 3a + 2 = 0$   
 $(a-2)(a-1) = 0$   
 $a-2=0 \Rightarrow a=2$  or  $a-1=0 \Rightarrow a=1$
- 66)  $x^2 - 12x + 35 = 0$   
 $(x-7)(x-5) = 0$   
 $x-7=0 \Rightarrow x=7$  or  $x-5=0 \Rightarrow x=5$
- 67)  $b^2 = 4b + 32$   
 $b^2 - 4b - 32 = 0$   
 $(b-8)(b+4) = 0$   
 $b-8=0 \Rightarrow b=8$  or  $b+4=0 \Rightarrow b=-4$
- 68)  $25x^2 - 16 = 0$   
 $(5x-4)^2 = 0 \Rightarrow 5x-4=0 \Rightarrow x=4/5$
- 69)  $7x^2 = 18x - 11$   
 $7x^2 - 18x + 11 = 0$   
 $(7x-11)(x-1) = 0$   
 $x=11/7$  or  $x=1$
- 70)  $8y^3 - 2y^2 = 0$   
 $2y^2(4y-1) = 0 \Rightarrow y=0$  or  $y=1/4$
- 71)  $4x^3 - 12x^2 + 8x = 0$   
 $4x(x^2 - 3x + 2) = 0$   
 $4x(x-2)(x-1) = 0$   
 $x=0$  or  $x=2$  or  $x=1$
- 72)  $9x^3 + 25x = 30x^2$   
 $9x^3 - 30x^2 + 25x = 0$   
 $x(9x^2 - 30x + 25) = 0$   
 $x(3x-5)^2 = 0 \Rightarrow x=0$  or  $x=5/3$

Sample for items 73-74:  $(a-1)(a+3) = 12$

$a^2 + 2a - 3 - 12 = 0$  expand the left side; bring over 12; set it = 0;

$(a-3)(a+5) = 0$  combine like terms; factor it;

$a = 3$  or  $a = -5$

- 73)  $(x+1)(x-5) = 16$   
 $x^2 - 4x - 5 = 16$   
 $x^2 - 4x - 21 = 0$   
 $(x-7)(x+3) = 0$   
 $x=7$  or  $x=-3$
- 74)  $(2z-5)(z-1) = 2$   
 $2z^2 - 7z + 5 = 2$   
 $2z^2 - 7z + 3 = 0$   
 $(2z-1)(z-3) = 0$   
 $z=1/2$  or  $z=3$

**Simplifying Fractions** - Follow the 2 examples below.

**Example 1:** Simplify  $\frac{3x+6}{3x+3y}$ .

**Solution:**  $\frac{3x+6}{3x+3y} = \frac{3(x+2)}{3(x+y)}$  Factor the numerator and denominator; look for common factors;  
 $= \frac{x+2}{x+y}$ ,  $x \neq -y$  Cancel out common factor which is 3;

**Example 2:** Simplify  $\frac{x^2-9}{(2x+1)(3-x)}$

**Solution:**  $\frac{x^2-9}{(2x+1)(3-x)} = \frac{(x+3)(x-3)}{-(2x+1)(x-3)}$  First factor the numerator; "pull out" a negative in the factor (3-x) to make it a common factor with the numerator;

$= -\frac{x+3}{2x+1}$ ,  $(x \neq -\frac{1}{2}, x \neq 3)$  exclude the first as it would make the denominator =0; exclude the 2<sup>nd</sup> as it would make both numerator and denominator =0.

Simplify. Give any restrictions on the variable.

75)  $\frac{5x-10}{x-2} = \frac{5(\cancel{x-2})}{\cancel{x-2}} = 5$

76)  $\frac{2a-4}{a^2-4} = \frac{2(a-2)}{(a+2)(a-2)} = \frac{2}{a+2}$   
 $a \neq \pm 2$   
 note!

77)  $\frac{(x-5)(2+7x)}{(5-x)(7x+2)}$   
 $= \frac{(\cancel{x-5})(2+7x)}{-(\cancel{x-5})(7x+2)}$   
 $= -1$   
 $x \neq 5, x \neq -2/7$

78)  $\frac{x^2+8x+16}{16-x^2} = \frac{(x+4)^2}{(4+x)(4-x)} = \frac{x+4}{4-x}$ ,  $x \neq \pm 4$

**Multiplying Fractions**

**Multiplication Rule for Fractions**

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$  To multiply fractions, you multiply their numerators and multiply their denominators.

Note: You can multiply first and then simplify, or you can simplify first and then multiply.

Example:  $\frac{x^2-x-12}{x^2-5x} \cdot \frac{x^2-25}{x+3} = \frac{(x-4)(\cancel{x+3})}{x(\cancel{x-5})} \cdot \frac{\cancel{(x-5)}(x+5)}{x+3}$   
 $= \frac{(x-4)(x+5)}{x}$  Notice how common factors were cancelled.

Simplify.

79)  $\frac{a+2}{a^2} \cdot \frac{3a}{a^2-4} = \frac{\cancel{a+2} \cdot 3\cancel{a}}{a^2(\cancel{a+2})(a-2)} = \frac{3}{a(a-2)}$

80)  $\frac{a^2-x^2}{a^2} \cdot \frac{a}{3x-3a} = \frac{(a+x)(a-x)}{a^2} \cdot \frac{a}{3(x-a)}$   
 $= \frac{-(a+x)(\cancel{x-a})}{a} \cdot \frac{1}{3(\cancel{x-a})}$   
 $= \left[ \frac{-(a+x)}{3a} \right] \text{ or } \left[ -\frac{a+x}{3a} \right]$

81) A triangle has base  $\frac{3x}{4}$  cm and height  $\frac{8}{9x}$  cm. What is its area?  
 $A = \frac{1}{2} \left( \frac{\cancel{3x}}{4} \right) \left( \frac{\cancel{8}}{9x} \right) = \frac{1}{3}$  Area =  $\frac{1}{2}(\text{base})(\text{height})$

**Dividing Fractions**

**Division Rule for Fractions:**  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$  Example:  $\frac{7}{9} \div \frac{2}{3} = \frac{7}{9} \cdot \frac{3}{2} = \frac{7}{6}$

To divide by a fraction, you multiply by its reciprocal!

$$87) \frac{2}{x^2-9} + \frac{4}{x+3} = \frac{2(x+3) + 4(x-3)}{(x-3)(x+3)} = \frac{2x+6+4x-12}{(x-3)(x+3)} = \frac{6x-6}{(x-3)(x+3)} =$$

$$= \frac{6(x-1)}{(x-3)(x+3)}$$

$$88) \frac{x}{x^2-1} + \frac{4}{x+1} = \frac{x+4(x-1)}{x^2-1} = \frac{5x-4}{x^2-1}$$

$$89) \frac{3a}{a-2b} + \frac{6b}{2b-a} = \frac{3a}{a-2b} - \frac{6b}{a-2b} = \frac{3a-6b}{a-2b} = \frac{3(a-2b)}{a-2b} = 3$$

$$90) \frac{x-11}{x^2-9} - \frac{x-7}{x^2-3x} = \frac{x-11}{(x+3)(x-3)} - \frac{x-7}{x(x-3)}$$

$$= \frac{x(x-11) - (x-7)(x+3)}{x(x+3)(x-3)}$$

$$= \frac{x^2-11x - (x^2-4x-21)}{x(x+3)(x-3)} = \frac{-7x+21}{x(x+3)(x-3)}$$

$$= \frac{-7(x-3)}{x(x+3)(x-3)} = \frac{-7}{x(x+3)}$$

$$83) \frac{x^2-1}{2} \div \frac{x+1}{16} = \frac{(x+1)(x-1)}{2} \cdot \frac{16}{x+1} = 8(x-1)$$

$$84) \frac{2a+2b}{a^2} \div \frac{a^2-b^2}{4a} = \frac{2(a+b)}{a^2} \cdot \frac{4a}{(a+b)(a-b)} = \frac{8}{a(a-b)}$$

$$85) \frac{4x^2-25}{x^2-16} \div \frac{12x+30}{2x^2+8x} = \frac{(2x+5)(2x-5)}{(x+4)(x-4)} \cdot \frac{2x(x+4)}{6(2x+5)} = \frac{2x(x+4)}{6(2x+5)}$$

$$= \frac{x(2x-5)}{3(x-4)}$$

Adding and Subtracting Algebraic Fractions

- Key Steps:** (1) Find the Least Common Denominator (LCD);  
 (2) Re-write each fraction being added or subtracted with the same common denominator.  
 (3) Add or subtract their numerators and write the result over the common denominator.

Example:

$$\frac{3}{6x-30} + \frac{8}{9x-45} = \frac{3}{6(x-5)} + \frac{8}{9(x-5)}$$

Factor out denominators to more easily identify LCD

$$= \frac{18(x-5)}{25} + \frac{18(x-5)}{16}$$

multiply the first fraction by 3, the second by 2

$$= \frac{18(x-5)}{25} \text{ or } \frac{18x-90}{25}$$

$$86) \frac{4x+3}{3} - \frac{7x}{4} + \frac{x-3}{6} = \frac{16x+12-21x+2x-6}{12} = \frac{-3x+6}{12} = \frac{-x+2}{4} \text{ or } \frac{2-x}{4}$$